

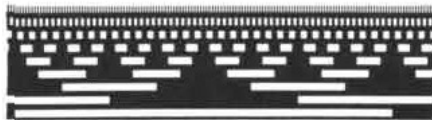
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# MATHEMATICAL GAMES

*Some old and new versions of ticktacktoe, plus the answers to last month's puzzles*

by Martin Gardner

Who has not as a child played ticktacktoe, that most ancient and universal struggle of wits of which Wordsworth wrote:

*At evening, when with pencil, and  
smooth slate  
In square divisions parcelled out  
and all  
With crosses and with cyphers  
scribbled o'er,  
We schemed and puzzled, head  
opposed to head  
In strife too humble to be named in  
verse.*

Forms of ticktacktoe were popular in ancient China, Greece and Rome—Ovid mentioned it in his *Art of Love*. At first sight it is not easy to understand the enduring appeal of a game which seems no more than child's play. While it is true that even in the simplest version of the game the number of possible moves is very large—15,120 ( $9 \times 8 \times 7 \times 6 \times 5$ ) different sequences for the first five moves alone—there are really only a few basic patterns, and any astute youngster can become an unbeatable player with only an hour or so of analysis of the game. But ticktacktoe also has its more complex variations and strategic aspects.

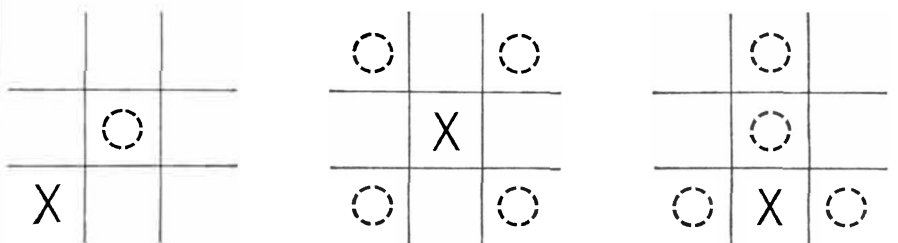
In the lingo of game theory, ticktacktoe is a two-person contest which is "finite" (comes to a definite end), has no element of chance and is played with "perfect information," all moves being known to both players. If played "ration-

ally" by both sides, the game must end in a draw. The only chance of winning is to catch an imperfect opponent in a "trap" where a row can be scored on the next move in two ways, only one of which can be blocked.

Of the three possible opening plays—a corner, the center or a side box—the strongest opening is the corner, because the opponent can avoid being trapped at the next move only by one of the eight possible choices: the center. The center opening can be met by seizing one of the four corners. The side opening is in many ways the most interesting, because of its richness in traps on both sides [see diagrams below].

A very ancient variant of the game gives each player three counters (one player may use pennies, the other dimes). The two players take turns placing a counter on the board until all six are down. If neither player has won by getting three in a row, each then is allowed to move one counter at a turn to an adjacent empty square, but he can move only vertically or horizontally, not diagonally [see diagram on page 162]. The first player has a sure win by placing his first counter in the center box, so this opening is usually barred. After any other opening, the second player must immediately take the center to avoid defeat. This game also ends in a draw with perfect play, but it swarms with potential traps on both sides.

There are variations of the game which permit diagonal moves (one of them attributed to early American Indians). A free-wheeling French version called "les pendus" (the hanged) allows any piece to be moved to any vacant



Three openings in ticktacktoe: corner (left), center (middle) and side (right)

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cell. This game is believed drawn if played rationally, whatever the opening.

The moving-counter game can be played with a matrix of 16 boxes (4 by 4), each player using four counters and striving to get four in a row. A few years ago the magician John Scarne marketed an interesting version, called "teeko," on a 5 by 5 board. The players have four counters and can move one space in any direction. The game is to get the four in a row (straight or diagonal) or to assemble them in a square formation on four adjacent cells.

There is a reverse version of simple ticktacktoe in which the first player to get three in a row *loses*. The second player has a decided advantage, but the first player can force a draw if he makes his initial move correctly. (I leave it to the reader to discover what the first move must be.)

In recent years several three-dimensional ticktacktoe games have been marketed. On a 3 by 3 by 3 cube the first player has an easy win, but a more complex version with a 4 by 4 by 4 cube is probably a draw if played rationally.

Four-dimensional ticktacktoe can be played on an imaginary hypercube cut into single-layer blocks [see diagram on page 165]. In the 4 by 4 by 4 by 4 version the object is to get four marks in a straight line on the hypercube. This is achieved by lining up four marks in a cube formed by piling up in serial order four blocks which occupy the same row or column or main diagonal. In this game the first player is believed to have a sure win, but in the 5 by 5 by 5 by 5 version the game probably must be a draw if played perfectly.

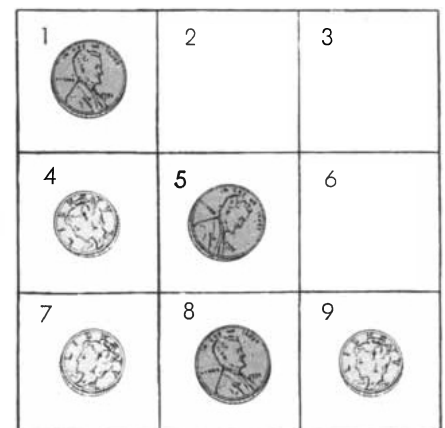
The ancient Japanese game of *gomoku* (five stones) is played on the intersections of a *go* board, which is equivalent to playing on a 19 by 19 square. Players take turns placing counters from an unlimited supply until one player wins by getting five in a line—vertical, horizontal or diagonal. No moves are allowed. The game became popular in England in the 1880s under the name of "gobang."

During the past decade a number of electrical machines for playing ticktacktoe have been constructed. Actually the first robot of this sort was invented by Charles Babbage, the early 19th-century originator of calculating machines [see "The Strange Life of Charles Babbage," by Philip and Emily Morrison; SCIENTIFIC AMERICAN, April, 1952]. Babbage planned to exhibit this machine in London to raise funds for more ambitious projects, but he never built it, because exhibits of other curious machines at the

time turned out to be financial failures. A novel feature of Babbage's robot was the fact that, when faced with a choice between equally rational lines of play, the machine made its selection on a random basis through a built-in mechanism, choosing one play if the number of games it had won up to that point was odd and the alternate play if the number was even. "An inquiring spectator," observed Babbage, "might watch a long time before he discovered the principle upon which it [the robot] acted."

It is not difficult to design a ticktacktoe machine (or program a digital computer) to play a rational game, but the problem becomes more complicated if the machine is to be designed to win the maximum number of games against inexperienced players. The difficulty lies in guessing how a novice is most likely to play. Just how shrewd will he be? To see the sort of complications that arise, let's consider the simple game, identifying the cells by number as on the board depicted below. Assume that the novice opens by taking cell number 8. The machine might do well to make an irrational response by seizing cell 3! This would be fatal against an expert, because the opponent of the machine has a sure win if he next takes cell 9, but an inexperienced player is not likely to hit upon this one winning reply. He will be strongly tempted, in fact, to take cell 4, because this leads to two promising traps against the robot. The machine, however, can now spring its own trap by taking cell 9, followed by 5 on the next move. In short, the machine may win more often with reckless strategies than with safe, rational lines of play, which are apt to lead to a draw.

A truly master player, robot or human, would not only know the most probable responses of novices in general but would also be able to analyze each individual opponent's style of play to de-



The game with moving counters

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*Here members of the Electronics Division discuss systems radar problems related to measurement of missile trajectories. Left to right: K. T. Larkin, radar and command guidance; Dr. S. B. Batdorf, head of the Electronics Division; Dr. R. J. Burke, telemetering; Dr. H. N. Leifer, solid state; S. Janken, product engineering.*

*Lockheed*

**MISSILE SYSTEMS DIVISION**

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termine what sort of mistakes he is likely to make, and in addition take account of the novice's improvement with increasing experience. At this point the humble game of ticktacktoe plunges us into far from trivial questions of probability and psychology.

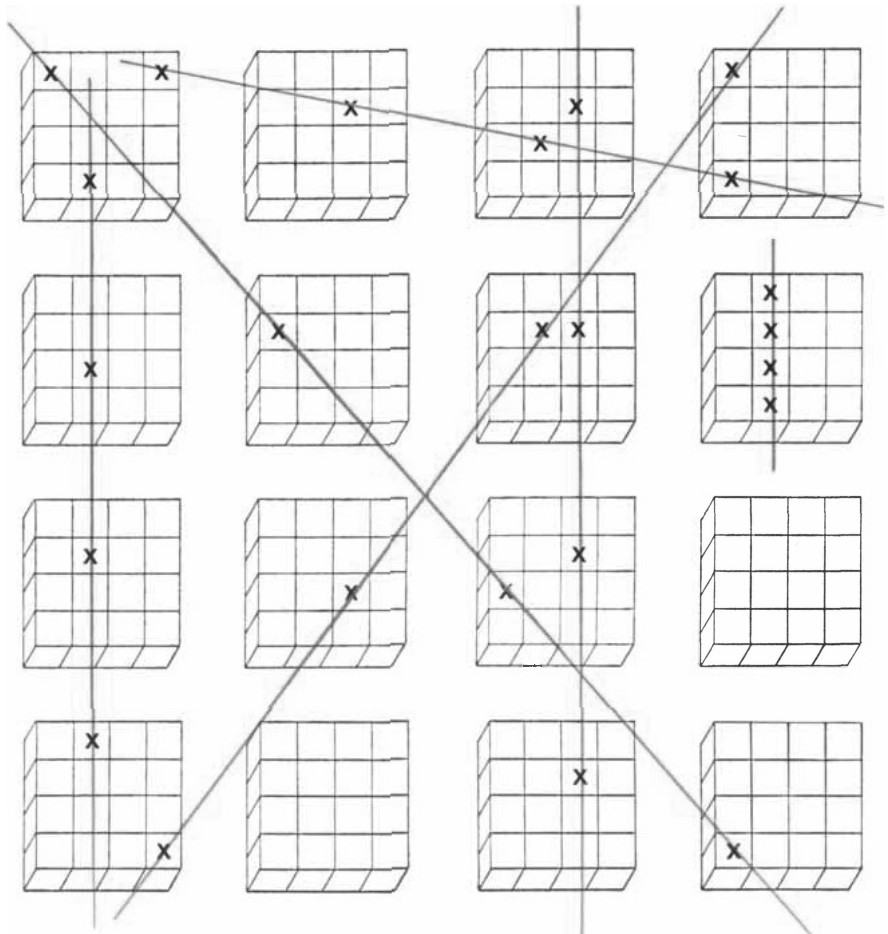
Following are the answers to the nine mathematical brain teasers published here last month.

1. Is there any other point on the globe, besides the North Pole, from which you could walk a mile south, a mile east and a mile north and find yourself back at the starting point? Yes indeed; not just one point but an infinite number of them! You could start from any point on a circle drawn around the South Pole at a distance slightly more than  $1 + 1/2\pi$  miles (about 1.16 miles) from the Pole—the distance is "slightly more" to take into account the curvature of the earth. After walking a mile south, your next walk of one mile east will take you on a complete circle around the Pole, and the walk one mile north from there will then return you to the starting point. Thus your starting point could be any one of the infinite number of points on the circle with a radius of about 1.16

miles from the South Pole. But this is not all. You could also start at points closer to the Pole, so that the walk east would carry you just twice around the Pole, or three times, or more, toward a limit of an infinite number of circlings of the Pole.

2. In the draw poker game with all cards showing, the first player wins every time if, and only if, he takes four 10s (his fifth card is immaterial) for his starting hand. This prevents his opponent from drawing any hand better than a nine-high straight flush from the rest of the deck. The first player, drawing to his original hand, can then keep one of his 10s and make a royal flush. If his opponent tries to forestall this by drawing four aces, say, the first player can draw a king-high, or at worst a 10-high, straight flush, beating four aces or four jacks, etc.

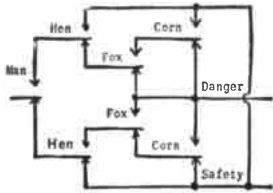
3. It is impossible to cover the mutilated chessboard (with two opposite corner squares cut off) with 31 dominoes, and the proof is easy. The two diagonally opposite corners are of the same color. Therefore their removal leaves a board with two more squares of one color than of the other. Each domino covers two squares of opposite color, since only op-



Four-dimensional ticktacktoe. Colored lines show some winning plays

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posite colors are adjacent. After you have covered 60 squares with 30 dominoes, you are left with two uncovered squares of the same color. These two cannot be adjacent, therefore they cannot be covered by the last domino.

4. The logician points to one of the roads and says to the native, "If I were to ask you if this road leads to the village, would you say 'yes'?" The native is forced to give the right answer, even if he is a liar! If the road does lead to the village, the liar would say "no" to the direct question, but as the question is put, he lies and says he would respond "yes." Thus the logician can be certain that the road does lead to the village, whether the respondent is a truth-teller or a liar. On the other hand, if the road actually does not go to the village, the liar is forced in the same way to reply "no" to the inquirer's question. More complicated forms of the question to the native can be devised, but they all hinge on the same logical principle: namely, that a double negative is equivalent to an affirmative.

5. You can learn the contents of all three boxes by drawing just one marble. The key to the solution is your knowledge that the labels on all three of the boxes are incorrect. You must draw a marble from the box labeled "black-white." Assume that the marble drawn is black. You know then that the other marble in this box must be black also, otherwise the label would be correct. Since you have now identified the box containing two black marbles, you can at once tell the contents of the box marked "white-white": you know it cannot contain two white marbles, because its label has to be wrong; it cannot contain two black marbles, for you have identified that box; therefore it must contain one black and one white marble. The third box, of course, must then be the one holding two white marbles. You can solve the puzzle by the same reasoning if the marble you draw from the "black-white" box happens to be white instead of black.

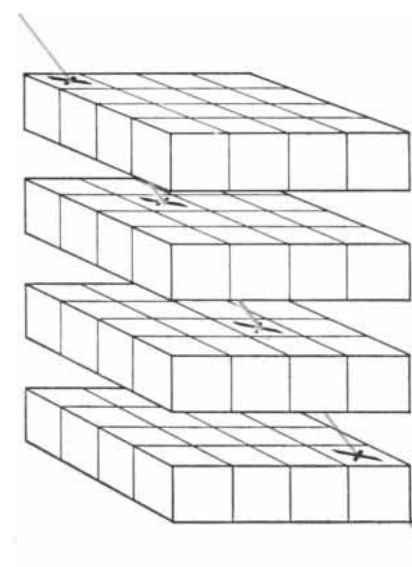
6. The answer to this puzzle is a simple matter of train schedules. While the Brooklyn and Bronx trains arrive equally often—at 10-minute intervals—it happens that their schedules are such that the Bronx train always comes to this platform one minute after the Brooklyn train. Thus the Bronx train will be the first to arrive only if the young man happens to come to the subway platform during this one-minute interval. If he enters the station at any other time—i.e., during a nine-minute interval—the Brooklyn train will come first. Since the young man's ar-

rival is random, the odds are nine to one for Brooklyn.

7. There is no way to reduce the cuts to fewer than six. This is at once apparent when you focus on the fact that a cube has six sides. The saw cuts straight—one side at a time. To cut the one-inch cube at the center (the one which has no exposed surfaces to start with) must take six passes of the saw.

8. The commuter has walked for 55 minutes before his wife picks him up. Since they arrive home 10 minutes earlier than usual, this means that the wife has chopped 10 minutes from her usual travel time to and from the station, or five minutes from her travel time to the station. It follows that she met her husband five minutes before his usual pick-up time of five o'clock, or at 4:55. He started walking at four, therefore he walked for 55 minutes. The man's speed of walking, the wife's speed of driving and the distance between home and station are not needed for solving the problem. If you tried to solve it by juggling figures for these variables, you probably found the problem aggravating.

9. The counterfeit stack can be identified by a single weighing of coins. You take one coin from the first stack, two from the second, three from the third and so on to the entire 10 coins of the tenth stack. You then weigh the whole sample collection on the pointer scale. The excess weight of this collection, in number of grams, corresponds to the number of the counterfeit stack. For example, if the group of coins weighs seven grams more than it should, then the counterfeit stack must be the seventh one, from which you took seven coins (each weighing one gram more than a genuine half-dollar).



A win on hypercube (see page 165)