

MATHEMATICAL GAMES

About mathematical games that are played on boards

by Martin Gardner

“Games possess some of the qualities of works of art,” Aldous Huxley has written. “With their simple and unequivocal rules, they are like so many islands of order in the vague untidy chaos of experience. When we play games, or even when we watch them being played by others, we pass from the incomprehensible universe of given reality into a neat little man-made world, where everything is clear, purposive and easy to understand. Competition adds to the intrinsic charm of games by making them exciting, while betting and crowd intoxication add, in their turn, to the thrills of competition.”

Huxley is speaking of games in general, but his remarks apply with special force to mathematical board-games in which the outcome is determined by pure thought, uncontaminated by physi-

cal prowess or the kind of blind luck supplied by dice, cards and other randomizing devices. Such games are as old as civilization and as varied as the wings of butterflies. Fantastic amounts of mental energy have been expended on them, considering the fact that until quite recently they had no value whatever beyond that of relaxing and refreshing the mind. Today they have suddenly become important in computer theory. Chess-playing and checker-playing machines that profit from experience may be the forerunners of electronic minds capable of developing powers as yet unimaginable.

The earliest records of mathematical board-games are found in the art of ancient Egypt, but they convey little information because of the Egyptian convention of showing scenes only in profile [see illustration below]. Some games involving boards have been found in Egyptian tombs [page 173], but they are not board games in the strict sense because they also involve a chance ele-

ment. A bit more is known about Greek and Roman board-games, but it was not until the 13th century A.D. that anyone thought it important enough to record the rules of a board game, and it was not until the 17th century that the first books on games were written.

Like biological organisms, games evolve and proliferate new species. A few simple games, such as ticktacktoe, may remain unchanged for centuries; others flourish for a time, then vanish completely. The outstanding example of a dinosaur diversion is rithmomachy. This was an extremely complicated number game played by medieval Europeans on a double chessboard with eight cells on one side and 16 cells on the other, and with pieces in the shapes of circles, squares and triangles. It traces back at least to the 12th century, and as late as the 17th century it was mentioned by Robert Burton, in *The Anatomy of Melancholy*, as a popular English game. Many learned treatises were written about it, but no one plays it today except a few mathematicians and medievalists.

In the U. S. the two most popular mathematical board-games are of course checkers and chess. Both have long and fascinating histories, with unexpected mutations in rules from time to time and place to place. Today the American checkers is identical with the English “draughts,” but in other countries there are wide variations. The so-called Polish checkers (actually invented in France)



Relief from a tomb at Sakkara in Egypt shows a board game in profile. Relief dates from 2500 B.C.



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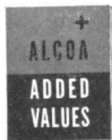


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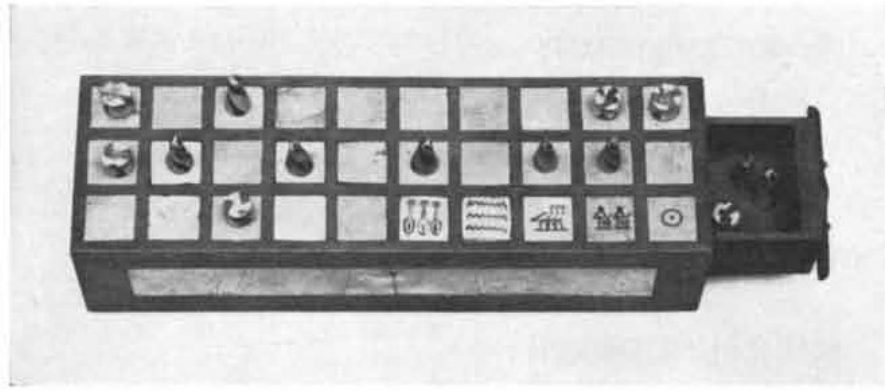
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Board game of senet, found in Egyptian tomb of 1400 B.C., also involved throwing sticks

is now the dominant form of the game throughout most of Europe. It is played on a 10-by-10 board, each side having 20 men that capture backward as well as forward. Crowned pieces (called queens instead of kings) move like the bishop in chess, and in making a jump can land on any vacant cell beyond the captured piece. The game is widely played in France (where it is called *dames*) and in Holland, and it is the subject of a large analytical literature. In the French-speaking provinces of Canada, and in parts of India, Polish checkers is played on a 12-by-12 board.

German checkers (*damenspiel*) resembles Polish checkers, but it is usually played on the English eight-by-eight board. A similar form of this "minor Polish" game, as it is sometimes called, is popular in the U.S.S.R., where it is known as *shashki*. Spanish and Italian variants also are closer to the English. Turkish checkers (*dama*) is also played on an eight-by-eight board, but each side has 16 men that occupy the second and third rows at the outset. Pieces move and jump forward and sideways, but not diagonally, and there are other radical departures from both the English and the Polish forms.

Chess likewise has varied enormously in its rules, tracing back ultimately to an unknown origin in India, probably in the sixth century A.D. True, there is 'oday an international chess that is standardized, but there are still many excellent non-European forms of the game that obviously share a common origin with international chess. Japanese chess (*sho-gi*) is played as enthusiastically in modern Japan as *go*, though only the latter game is known in Western countries. *Sho-gi* is played on a nine-by-nine board, with 20 men on each side, arranged at the start on the first three rows. The game is won, as in Western chess, by checkmating a piece that moves exactly like the king. An inter-

esting feature of the game is that captured pieces can be returned to the board to be used by the captor.

Chinese chess (*siang k'i*) also ends with the checkmate of a piece that moves like the king in Western chess, but the rules are quite different from those of Japanese chess, and its eight-by-eight board is divided across the center by a blank horizontal row called "the river." Martian chess ("jetan"), explained by Edgar Rice Burroughs in the appendix to his novel *The Chessmen of Mars*, is a surprisingly well-thought-out variant, played on a 10-by-10 board with unusual pieces and novel rules. For example, the princess (which corresponds roughly to our king) has the privilege of one "escape move" per game that permits her to flee an unlimited distance in any direction.

In addition to these regional variants of chess, modern players, momentarily bored with the orthodox game, have invented a weird assortment of games known as fairy chess. Among the many fairy-chess games that can be played on the standard board are: two-move chess, in which each player plays twice on his turn; a game in which one side plays with no pawns, or with an extra row of pawns instead of a queen; cylindrical chess, in which the left side of the board is considered joined to the right side (if the board is thought of as having a half-twist before the sides are joined, it is called Moebius-strip chess); transportation chess, in which any piece can be moved on top of the rook and carried by the rook to another square. Dozens of strange new pieces have been introduced, such as the chancellor (combining the moves of rook and knight), the centaur (combining bishop and knight) and even neuter pieces (e.g., a blue queen) that can be played by either side. (In Lewis Padgett's science-fiction novel *The Fairy Chessmen* a war is won by a mathematician who makes a hobby

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of fairy chess. His mind, accustomed to breaking rules, is elastic enough to cope with an equation too bizarre for his more brilliant but more orthodox colleagues.)

An amusing species of fairy chess that is quite old, but still provides a delightful interlude between more serious games, is played as follows. One player sets up his 16 men in the usual way, but his opponent has only one piece, called the maharajah. A queen may be used for this piece, but its moves combine those of queen and knight. It is placed at the outset on any free square not threatened by a pawn; then the other side makes the first move. The maharajah loses if he is captured, and wins if he checkmates the king. It might be thought that the maharajah has a poor chance of winning, but his mobility is so great that if he moves swiftly and aggressively, he often checkmates early in the game. At other times he can sweep the board clean of pieces and then force the lone king into a corner checkmate.

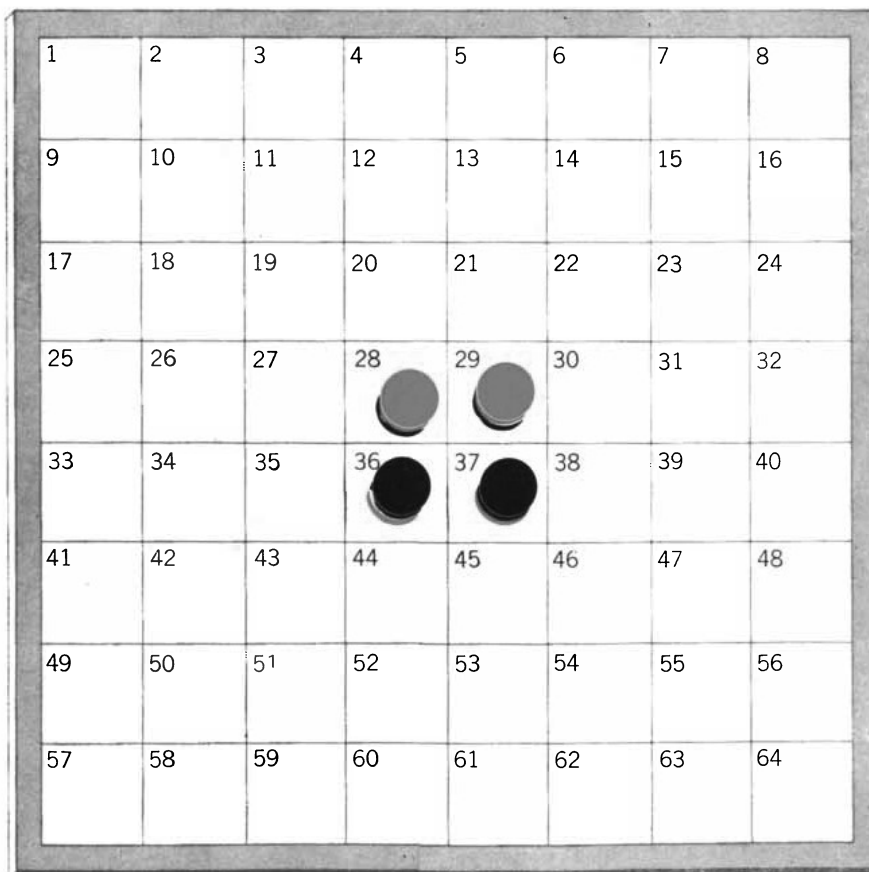
Hundreds of games have been invented that are played on a standard chessboard but have nothing in common with either chess or checkers. One of the best, in my opinion, is the now-forgotten game of "reversi." It uses 64 counters that have contrasting colors, say red and

black, on their opposite sides. A crude set can be made by coloring one side of a sheet of cardboard, then cutting out small circles; a better set can be constructed by buying six boxes of inexpensive checkers and gluing the pieces into red-black pairs. It is worth the trouble, because the game can be an exciting one for every member of the family.

Reversi starts with an empty board. One player has 32 pieces turned red-side up; the other has 32 turned black-side up. Players alternate in placing a single man on the board in conformity with the following rules:

1. The first four men must be placed on the four central squares. Experience has shown that it is better for the first player to place his second man above, below, or to the side of his first piece (an example is shown in the illustration below), rather than diagonally adjacent, but this is not obligatory.

2. After the four central squares are filled, players continue placing single pieces. Each must be placed so that it is adjacent to a hostile piece, orthogonally or diagonally. Moreover, it must also be placed so that it is in direct line with another piece of the same color, and with one or more enemy pieces (and no vacant cells) in between. In other words,



An opening for the board game of reversi. Numbers are for reference only



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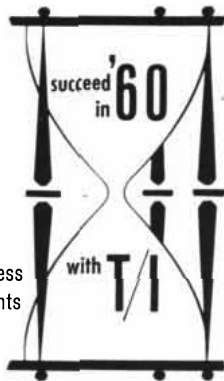
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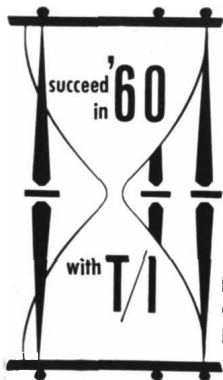
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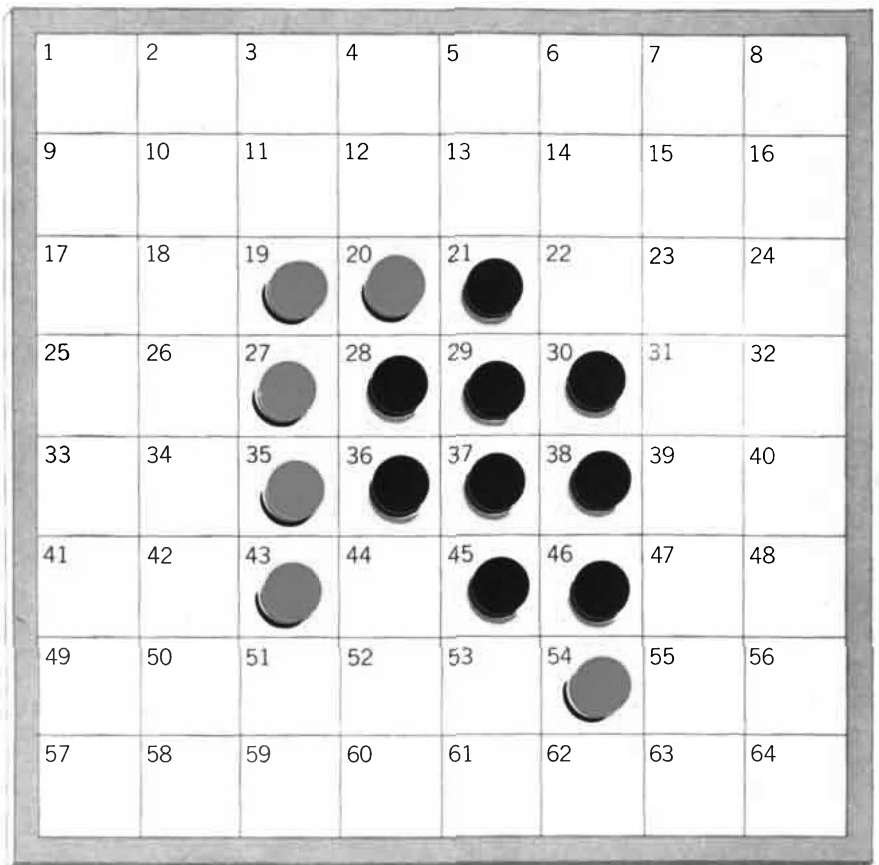
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If reversi player with colored pieces makes the next move, he can win six pieces

a piece must always be placed so that it is one of a pair of friendly pieces on opposite sides of an enemy piece or at opposite ends of a chain of enemy pieces. The enemy pieces are considered captured, but instead of being removed they are turned over, or "reversed," so that they become friendly pieces. Pieces remain fixed throughout the game, but may be reversed any number of times.

3. If the placing of a piece simultaneously captures more than one chain of enemy pieces, the pieces in both chains are reversed.

4. Pieces are captured only by the placing of a hostile piece. Chains that become flanked at both ends as a result of other causes are not captured.

5. If a player cannot move, he loses his turn. He continues to lose his turn until a legal move becomes possible for him.

6. The game ends when all 64 squares are filled, or when neither player can move (either because he has no legal move or because his counters are gone). The winner is the person with the most pieces on the board.

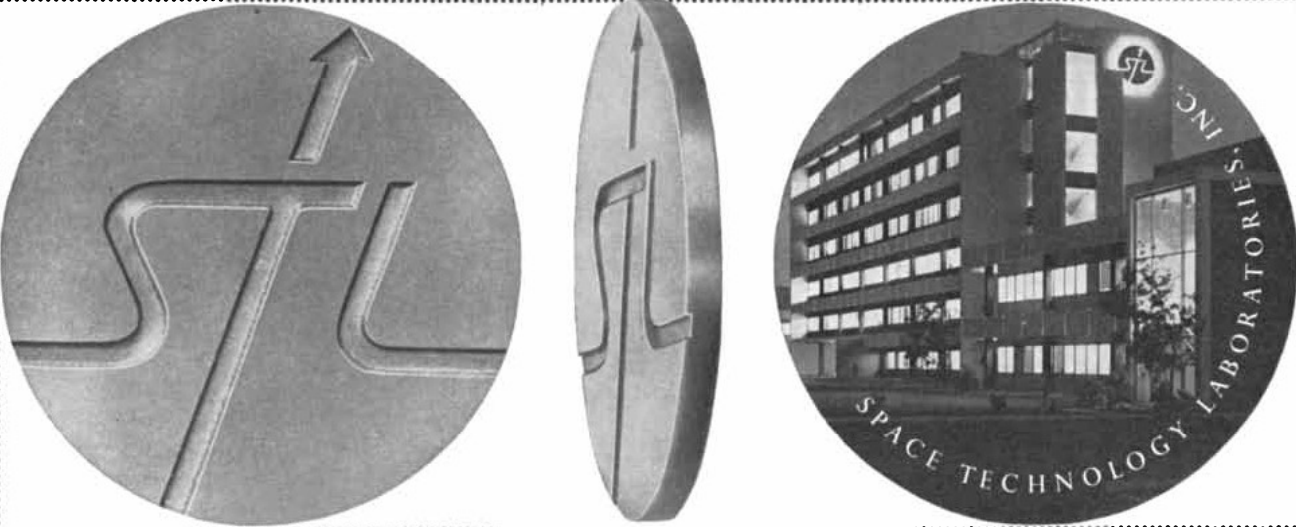
Two examples will clarify the rules: In the illustration on page 174 red can play only on cells 43, 44, 45 and 46. In each case he captures and reverses a

single piece. In the illustration on this page, if red plays on cell 22 he is compelled to reverse six pieces: 21, 29, 36, 30, 38 and 46. As a result the board, which formerly was mostly black, suddenly becomes mostly red. Dramatic reversals of color are characteristic of this unusual game, and it is often difficult to say who has the better game until the last few plays are made. The player with the fewest pieces frequently has a strong positional advantage.

Some pointers for beginners: If possible, confine early play to the central 16 squares, and try especially to occupy cells 19, 22, 43 and 46. The first player forced outside this area is usually placed at a disadvantage. Outside the central 16 squares, the most valuable cells to occupy are the corners of the board. For this reason it is unwise to play on cells 10, 15, 50 or 55, because this gives your opponent a chance to take the corner cells. Next to the corners, the most desirable cells are those that are next but one to the corners (3, 6, 17, 24, 41, 48, 59 and 62). Avoid giving your opponent a chance to occupy these cells. Deeper rules of strategy will occur to any player who advances beyond the novice stage.

Little in the way of analysis has been published about reversi; it is hard to say

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who, if either player, has the advantage on even a board as small as four-by-four. Here is an interesting problem that some readers may enjoy trying to solve before the answer is given in this department next month. Is it possible for a game to occur in which a player, before his 10th move, wins by removing *all* the enemy pieces from the board?

An amusing thing about the history of reversi is that two Englishmen, Lewis Waterman and John W. Mollett, both claimed to be the sole inventor. Each called the other a fraud. In the late 1880's, when the game was enormously popular in England, rival handbooks and rival firms for the manufacture of equipment were authorized by the two claimants. Regardless of who invented it, reversi is a game that combines complexity of structure with rules of delightful simplicity, and a game that does not deserve oblivion.

The answer in smallest numbers for last month's Lewis Carroll problem of finding three right triangles with integral sides and equal areas is 40, 42 and 58; 24, 70 and 74; and 15, 112 and 113. In each case the area is 840. Had Carroll doubled the size of the two triangles that he found, he would have obtained the first two triangles cited above, from which the step to the third would have been easy. Henry Ernest Dudeney, in the answer to problem 107 in his *Canterbury Puzzles*, gives a formula by which such triangle triplets can be easily found.

Carroll's truth-and-lie problem has only one answer that does not lead to a logical contradiction: A and C lie; B speaks the truth. The problem yields easily to the propositional calculus by taking the word "says" as the logical connective called equivalence. Without drawing on symbolic logic one can simply list the eight possible combinations of lying and truth-telling for the three men, then explore each combination, eliminating those that lead to logical contradictions.

Carroll's solutions to the six doublets are: Grass, crass, cress, tress, trees, frees, freed, greed, green; ape, are, err, ear, mar, man; one, owe, ewe, eye, dye, doe, toe, too, two; blue, glue, glut, gout, pout, port, part, pant, pint, pink; winter, winner, wanner, wander, warder, harder, harper, hamper, damper, damped, dammed, dimmed, dimmer, simmer, summer; rouge, rough, sough, south, sooth, booth, boots, boats, brats, brass, crass, cress, crest, chest, cheat, cheap, cheep, cheek.

The letters *abcdefghi* rearrange to make the hyphenated word *big-faced*.