MATHEMATICAL GAMES

The game of solitaire and some variations and transformations

by Martin Gardner

"The game called solitaire pleases me much," the great German mathematician Gottfried von Leibniz wrote in a letter in 1716. "I take it in reverse order. That is to say, instead of making a figure according to the rules of the game, which is to jump to an empty place and remove the piece over which one has jumped, I thought it better to reconstruct what had been demolished, by filling an empty hole over which one has leaped. In this way one may set oneself the task of forming a given figure if that is possible, as it certainly is if it can be destroyed.

"But why all this? you ask. I reply: to perfect the art of invention. For we must have the means of constructing everything which is found by the exercise of reason."

Leibniz' last two sentences are a bit obscure. Perhaps they mean that it is worthwhile to analyze everything that has a logical or mathematical structure.

Worthwhile or not, no other puzzle game played on a board with counters has enjoyed such a long, uninterrupted run of popularity as solitaire. Its origin is unknown, although its invention is sometimes attributed to a prisoner in the Bastille. That it was widely played in France during the late 19th century is evident from the many French books and articles that were then written about the game. It is likely that almost every reader of this column has at one time or another racked his brain over the puzzle. At present several versions of solitaire are on sale in this country under various trade names, some with pegs that are moved from hole to hole and some with marbles that rest in circular depressions. The marble versions are easier to manipulate. One can also play by placing pennies, beans, small poker chips or any other type of counter on the board depicted on page 158.

This board, which has 33 cells, is the most popular form of solitaire in England, the U.S. and the U.S.S.R. In France the board has four additional cells at the positions indicated by the four dots. Both forms of the board are found throughout the rest of western Europe. The cells are labeled in traditional fashion, the first digit of each number giving the position of the column from left to right, the second digit giving the position of the row from bottom to top.

The basic problem—usually the only problem supplied by manufacturers of the puzzle—begins with counters placed on all cells except the center one. The object is to make a series of jumps that will remove every counter but one. For an elegant solution this last counter should be left on the central cell. A "jump" consists of moving a counter over any adjacent counter to land on the next empty cell. The jumped counter is taken off the board. This is the same as a jump in checkers except that each jump must be straight to the left or right, or straight up or down. No diagonal jumps are allowed.

Each move must be a jump. If a point is reached at which no jumps are possible, the game ends in a stalemate. A single piece may continue in a chain of connected jumps as long as jumps are available, but it need not do so. A chain of jumps is counted as a single "move." To solve the puzzle 31 jumps obviously must be made, but if some are in chains, the number of moves can be fewer.

No one knows how many different ways there are to solve the puzzle leaving the last counter in the center. Scores of solutions have been published. Before discussing some of them, however, readers unfamiliar with solitaire are urged to try the six simpler figures shown in the illustration on page 160. In each case the last counter must be left on the center cell. For example, the Latin cross is easily solved in five moves: 45–25, 43–45, 55–35, 25–45, 46–44.

After mastering these traditional problems the reader may want to try the three puzzles shown on page 162. In

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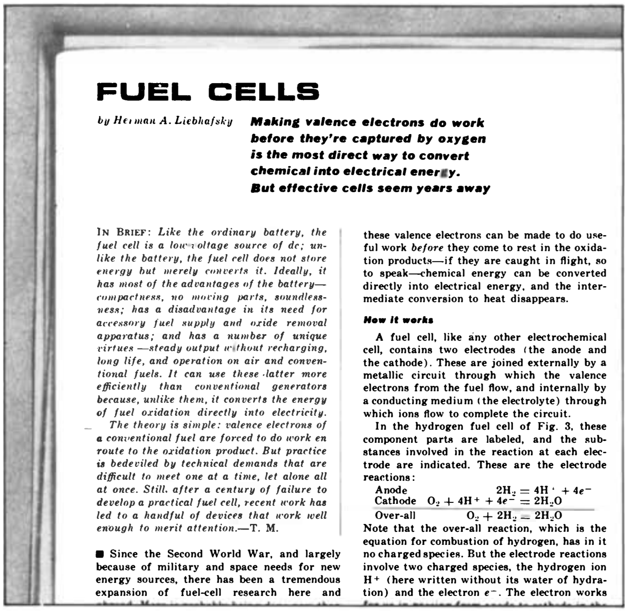


FUEL CELLS

by Herman A. Liebhafsky

Making valence electrons do work before they're captured by oxygen is the most direct way to convert chemical into electrical energy. But effective cells seem years away

IN BRIEF: *Like the ordinary battery, the fuel cell is a low-voltage source of dc; unlike the battery, the fuel cell does not store energy but merely converts it. Ideally, it has most of the advantages of the battery—compactness, no moving parts, soundlessness; has a disadvantage in its need for accessory fuel supply and oxide removal apparatus; and has a number of unique virtues—steady output without recharging, long life, and operation on air and conventional fuels. It can use these latter more efficiently than conventional generators because, unlike them, it converts the energy of fuel oxidation directly into electricity.*

The theory is simple: valence electrons of a conventional fuel are forced to do work en route to the oxidation product. But practice is bedeviled by technical demands that are difficult to meet one at a time, let alone all at once. Still, after a century of failure to develop a practical fuel cell, recent work has led to a handful of devices that work well enough to merit attention.—T. M.

■ Since the Second World War, and largely because of military and space needs for new energy sources, there has been a tremendous expansion of fuel-cell research here and

these valence electrons can be made to do useful work *before* they come to rest in the oxidation products—if they are caught in flight, so to speak—chemical energy can be converted directly into electrical energy, and the intermediate conversion to heat disappears.

How it works

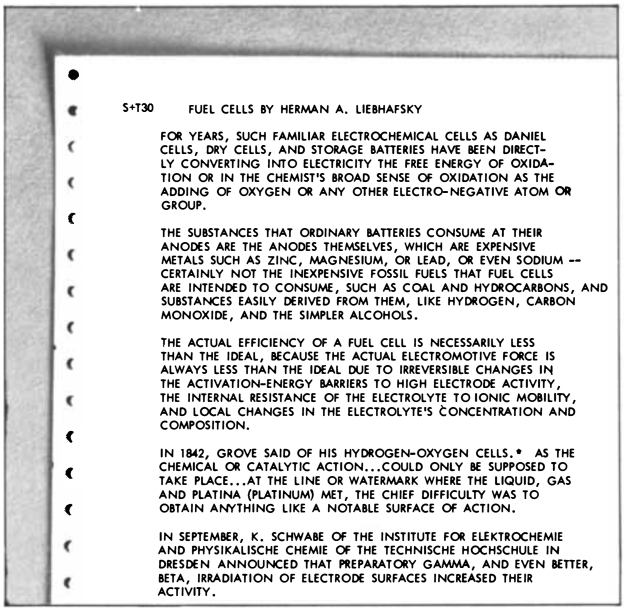
A fuel cell, like any other electrochemical cell, contains two electrodes (the anode and the cathode). These are joined externally by a metallic circuit through which the valence electrons from the fuel flow, and internally by a conducting medium (the electrolyte) through which ions flow to complete the circuit.

In the hydrogen fuel cell of Fig. 3, these component parts are labeled, and the substances involved in the reaction at each electrode are indicated. These are the electrode reactions:

$$\begin{array}{ll} \text{Anode} & 2H_2 = 4H^+ + 4e^- \\ \text{Cathode} & O_2 + 4H^+ + 4e^- = 2H_2O \\ \hline \text{Over-all} & O_2 + 2H_2 = 2H_2O \end{array}$$

Note that the over-all reaction, which is the equation for combustion of hydrogen, has in it no charged species. But the electrode reactions involve two charged species, the hydrogen ion H^+ (here written without its water of hydration) and the electron e^-. The electron works

This 4000-word article appeared in the January, 1962, issue of International Science and Technology. *To abstract the article, a document analyst would read it, define its purpose, and summarize its essential points.*



S+T30 FUEL CELLS BY HERMAN A. LIEBHAFSKY

FOR YEARS, SUCH FAMILIAR ELECTROCHEMICAL CELLS AS DANIEL CELLS, DRY CELLS, AND STORAGE BATTERIES HAVE BEEN DIRECTLY CONVERTING INTO ELECTRICITY THE FREE ENERGY OF OXIDATION OR IN THE CHEMIST'S BROAD SENSE OF OXIDATION AS THE ADDING OF OXYGEN OR ANY OTHER ELECTRO-NEGATIVE ATOM OR GROUP.

THE SUBSTANCES THAT ORDINARY BATTERIES CONSUME AT THEIR ANODES ARE THE ANODES THEMSELVES, WHICH ARE EXPENSIVE METALS SUCH AS ZINC, MAGNESIUM, OR LEAD, OR EVEN SODIUM -- CERTAINLY NOT THE INEXPENSIVE FOSSIL FUELS THAT FUEL CELLS ARE INTENDED TO CONSUME, SUCH AS COAL AND HYDROCARBONS, AND SUBSTANCES EASILY DERIVED FROM THEM, LIKE HYDROGEN, CARBON MONOXIDE, AND THE SIMPLER ALCOHOLS.

THE ACTUAL EFFICIENCY OF A FUEL CELL IS NECESSARILY LESS THAN THE IDEAL, BECAUSE THE ACTUAL ELECTROMOTIVE FORCE IS ALWAYS LESS THAN THE IDEAL DUE TO IRREVERSIBLE CHANGES IN THE ACTIVATION-ENERGY BARRIERS TO HIGH ELECTRODE ACTIVITY, THE INTERNAL RESISTANCE OF THE ELECTROLYTE TO IONIC MOBILITY, AND LOCAL CHANGES IN THE ELECTROLYTE'S CONCENTRATION AND COMPOSITION.

IN 1842, GROVE SAID OF HIS HYDROGEN-OXYGEN CELLS.* AS THE CHEMICAL OR CATALYTIC ACTION...COULD ONLY BE SUPPOSED TO TAKE PLACE...AT THE LINE OR WATERMARK WHERE THE LIQUID, GAS AND PLATINA (PLATINUM) MET, THE CHIEF DIFFICULTY WAS TO OBTAIN ANYTHING LIKE A NOTABLE SURFACE OF ACTION.

IN SEPTEMBER, K. SCHWABE OF THE INSTITUTE FOR ELEKTROCHEMIE AND PHYSIKALISCHE CHEMIE OF THE TECHNISCHE HOCHSCHULE IN DRESDEN ANNOUNCED THAT PREPARATORY GAMMA, AND EVEN BETTER, BETA, IRRADIATION OF ELECTRODE SURFACES INCREASED THEIR ACTIVITY.

This abstract was prepared by an IBM computer. The text was first coded in machine language. The computer then counted key words, and printed out sentences having the greatest statistical significance.

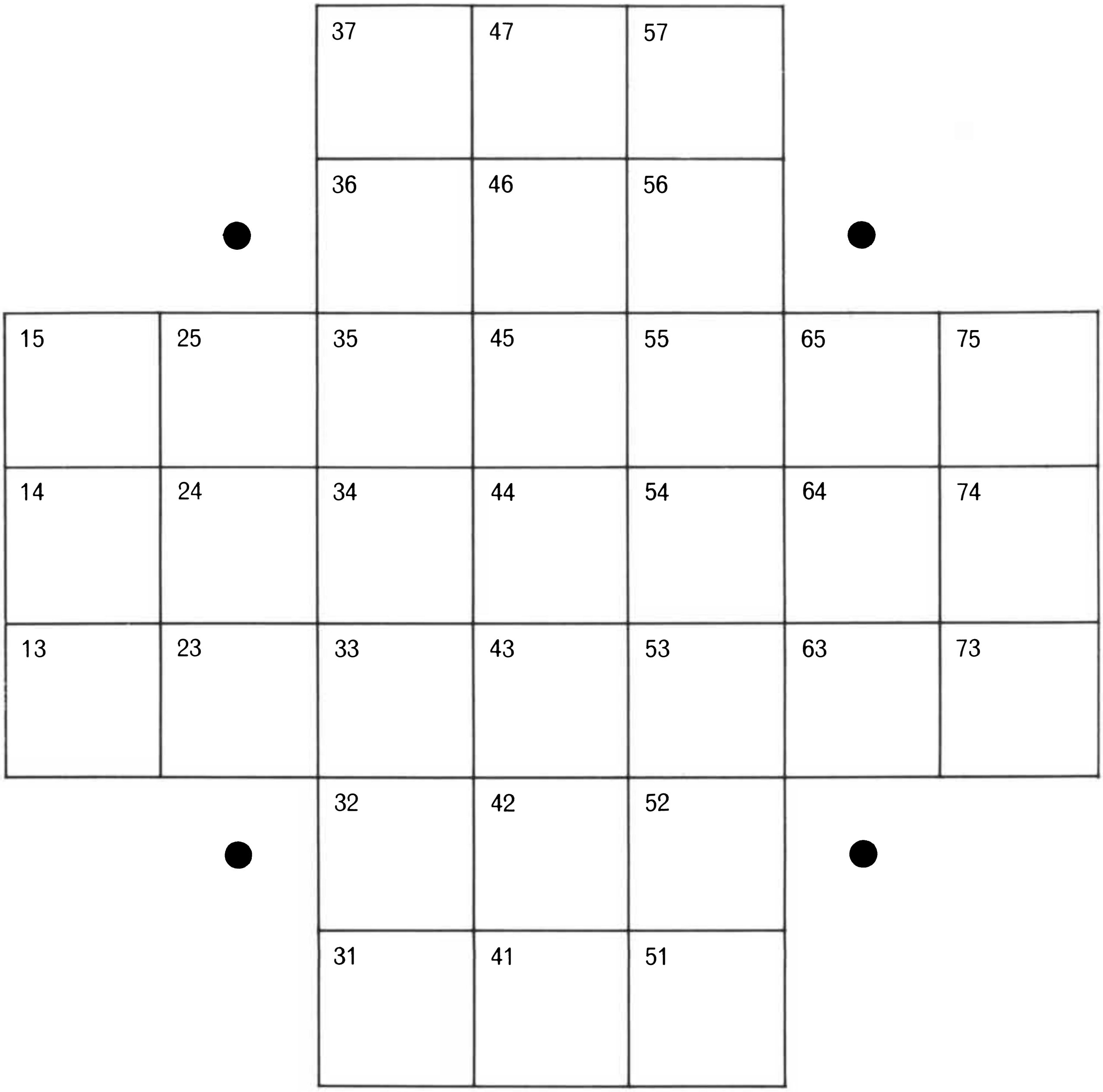
Each year in the physical and life sciences, some 50,000 technical journals will be published throughout the world. 100,000 research reports and 60,000 technical books will also be written. Somewhere in this mass of knowledge may be information you need. To tell what is known—and where to find it—IBM is investigating systems for the dissemination, storage, and retrieval of information.

To create an advanced information retrieval system, labels must be found for *all* useful information in documents. With conventional library indexing, it is difficult to make allowance for new kinds of knowledge. However, computers let us use more versatile methods of indexing. In one of these, the KWIC INDEX (Key Word In Context), a computer selects significant terms in the titles of documents, then prints them out as index entries.

Once indexed, characteristics of documents' contents can be used to notify people of their existence. The Selective Dissemination of Information system at IBM stores profiles describing individuals' interests. A new document's key words are matched against key words in a person's profile. If there is sufficient correlation, he is informed of the document. Profile matching can also be used to retrieve information by storing documents and feeding key-word queries through the system.

At present it is relatively difficult to get text into machine-readable form. However, the development of high-speed optical character readers, automatic language translators, and improved methods of capturing linguistic information at the source may make it possible to introduce information directly into retrieval systems. Once harvested, vast quantities of information will present storage problems. IBM is investigating random-access photostorage systems capable of storing millions of documents and retrieving them in seconds. Out of systems like these may come total information centers which will acquaint scientists and businessmen with all the information needed in their work.

If you have been searching for an opportunity to make important contributions in information retrieval, component engineering, optics, space systems, or any of the other fields in which IBM scientists and engineers are finding answers to basic questions, please contact us. IBM is an Equal Opportunity Employer. Write to: Manager of Professional Employment, IBM Corporation, Department 659S, 590 Madison Avenue, New York 22, New York.



The solitaire board

each of these one must begin with a full board, except for a vacant center cell, and play until the figure shown remains on the board. The first puzzle is easy; the other two are not. Note that the pinwheel is a stalemated position. (It is possible to reach a stalemate in as few as six moves. Can you discover how? The answer to this and the other problems will appear in "Mathematical Games" next month.)

Advanced students of solitaire have gone to fantastic lengths in setting themselves unusual tasks. For example, in his book *The Game of Solitaire* (1920) Ernest Bergholt introduces into his brilliant problems a variety of curious restrictions. (All the problems start with a full board, although the vacant cell need not be in the center.) His "ball on the watch" is a single counter—preferably a different color from the others—that must not be moved until the end of the game; then it captures one or more pieces to become the sole survivor. His "dead ball" is a counter that remains untouched throughout and is the last to be taken. A "sweep" is a long chain of jumps that closes a game. Bergholt gives many examples of games ending in eight-ball sweeps. It is possible, he maintains, to begin with the vacancy at 37 and end with a nine-ball sweep.

What is the smallest number of moves required to reduce a full board of 32 pieces to a single piece? If the initial vacancy is 13, 23 or 33 (or any of the other 17 cells that correspond to these three when the board is rotated or reflected), all counters but one can be removed in 16 moves. This is the shortest possible solution. If the vacancy is on any other cell, 17 is thought to be the minimum.

If the game opens with an empty cen-

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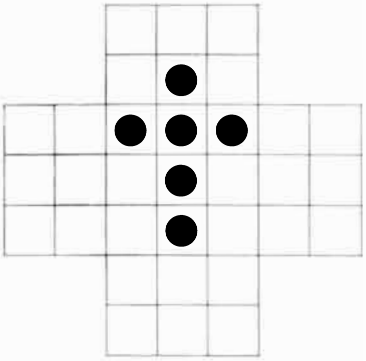
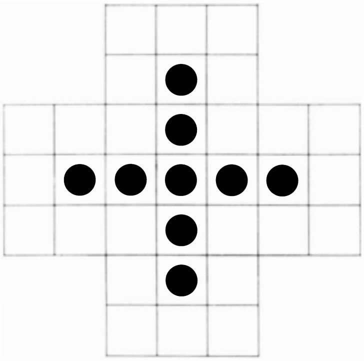
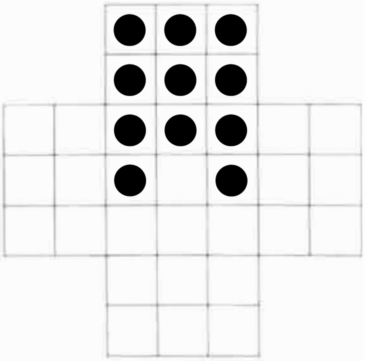
ter cell and ends with a counter on the same cell, 18 moves are required. Henry Ernest Dudeney, in his *Amusements in Mathematics* (Problem No. 227), gives a 19-move solution and adds: "I do not think the number of moves can be reduced." But Bergholt gives in his book the following 18-move solution: 46–44, 65–45, 57–55, 54–56, 52–54, 73–53, 43–63, 75–73–53, 35–55, 15–35, 23–43–63–65–45–25, 37–57–55–53, 31–33, 34–32, 51–31–33, 13–15–35, 36–34–32–52–54–34, 24–44.

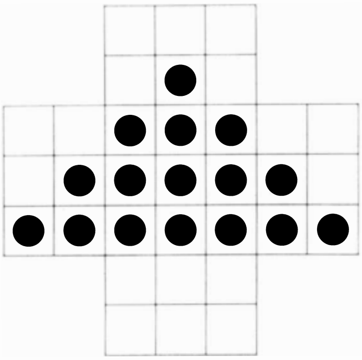
"I will venture to assert," writes Bergholt, "that this record will never be beaten." Perhaps some reader can do to Bergholt what Bergholt did to Dudeney. Note that if the chain of jumps in Bergholt's next-to-last move is not interrupted, a 17-move solution is achieved, ending on cell 14, with the counter originally placed on cell 36 serving as a ball on the watch that closes the game with a six-ball sweep.

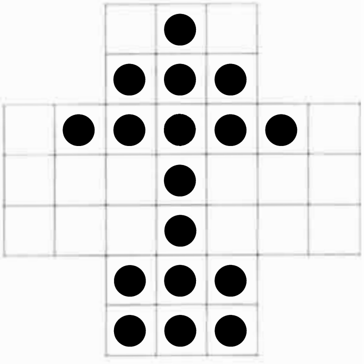
Other solutions of the classic center-to-center problem, although failing to


LATIN CROSS


GREEK CROSS


FIREPLACE


PYRAMID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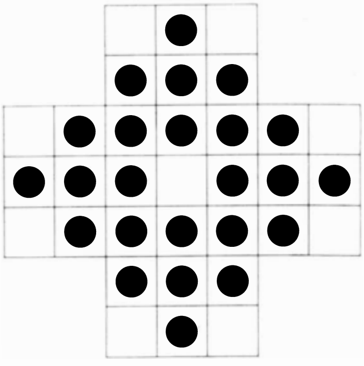
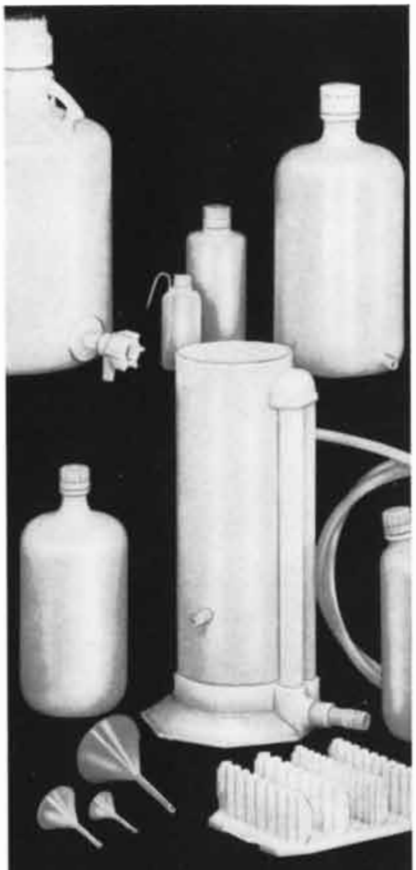
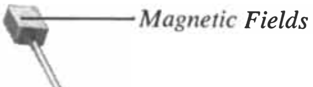
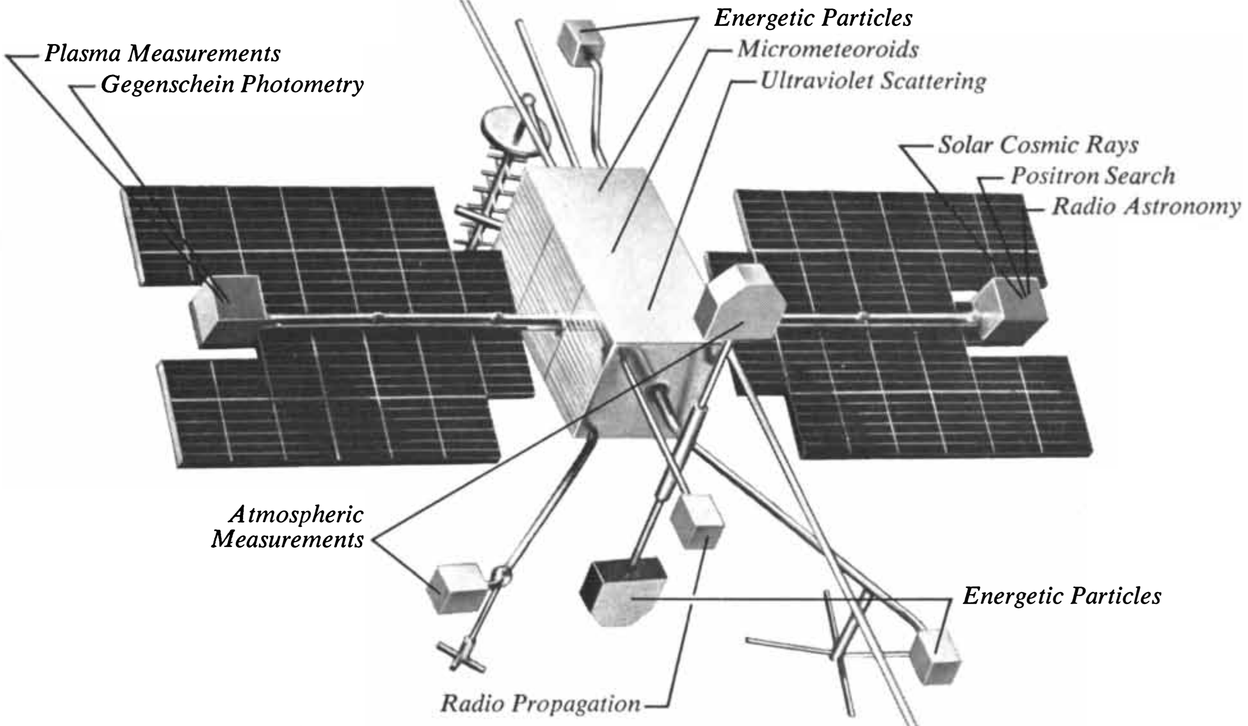

INCLINED SQUARE

Traditional problems in which the last counter is to be left in the center





OGO: its first mission. Sometime in 1963, OGO (NASA's Orbiting Geophysical Observatory) will be launched into an elliptical orbit around the earth. It will gather, process and transmit data on the physics of near-earth and cislunar space. Here are some of the studies OGO may undertake in this initial flight: *Energetic particles,* with nine separate experiments on the flux and characteristics of these particles (including cosmic ray and plasma studies). *Radio propagation and astronomy,* through measurements of ambient radio energy not accessible from earth. *Micrometeoroids,* to determine the mass distribution and direction of interplanetary dust in the vicinity of earth. *Magnetic fields,* their intensity, direction and variation near earth and in space. *Atmospheric measurements,* to study the pressure, temperature and composition of earth and cislunar space. *Ultraviolet scattering,* from hydrogen in space. *Gegenschein photometry,* to study sunlight scattered by interplanetary matter. OGO will be launched into a wide range of orbits and may carry as many as 50 different experiments on each of its missions. This Orbiting Geophysical Observatory will be one of the most versatile earth satellites man has ever built.



**Captions indicate possible arrangement of instrumentation clusters which OGO may carry.*

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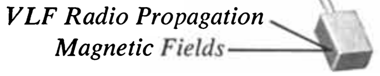

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Los Angeles • Vandenberg AFB • Norton AFB, San Bernardino • Cape Canaveral • Washington, D.C. • Boston • Huntsville • Dayton, achieve the minimum in moves, often have a remarkable symmetry. The following three are taken from *Puzzle Craft*, a booklet edited by Lynn Rohrbough and published in 1930 by the Co-operative Recreation Service of Delaware, Ohio.

"The Fireplace" (discovered by Josephine G. Richardson of Boston): 42–44, 63–43, 44–42, 51–53, 41–43–63, 73–53, 75–73, 65–63, 54–52, 73–53, 52–54, 23–43, 31–33, 43–23, 13–33, 15–13, 25–23, 34–32, 13–33, 32–34. The counters now form the fireplace shown in the illustration on page 160. The game is completed according to the solution of that puzzle.

"The Six-Ball Sweep": 46–44, 65–45, 57–55, 37–57, 54–56, 57–55, 52–54, 73–53, 75–73, 43–63, 73–53, 23–43, 31–33, 51–31, 34–32, 31–33, 36–34, 15–35, 13–15, 45–25, 15–35. The pattern now has vertical symmetry. A six-ball sweep (43–63–65–45–25–23–43) reduces the pattern to a T figure, easily solved with 44–64, 42–44, 34–54, 64–44.

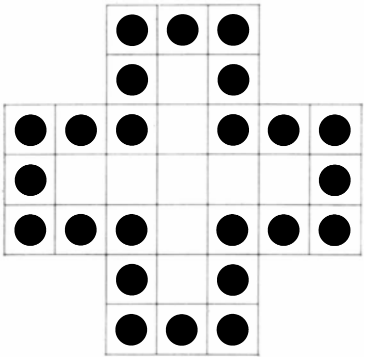
"The Jabberwocky": 46–44, 65–45, 57–55, 45–65, 25–45, 44–46, 47–45, 37–35, 45–25. The pattern is vertically symmetrical. The next 16 moves are mirror-image pairs that can be made simultaneously by the right and left hands, as follows:

Left hand	Right hand
15–35	75–55
34–36	54–56
14–34	74–54
33–35	53–55
36–34	56–54
31–33	51–53
34–32	54–52
13–33	73–53

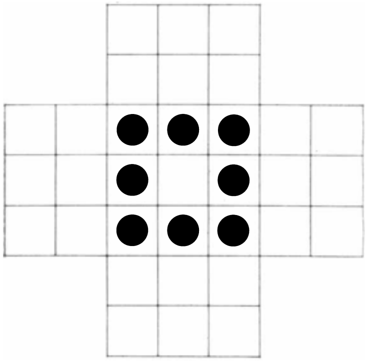
The solution concludes: 43–63, 33–31–51–53, 63–43, 42–44.

The mathematical theory behind solitaire is only partly known. In fact, one of the major unsolved problems of recreational mathematics is finding a way to analyze a given solitaire position to determine whether or not it is possible to reduce it to another given position. The man who has made the most progress in this direction is Mannis Charosh, a teacher of mathematics at New Utrecht High School in Brooklyn, N.Y. In *The Mathematics Student Journal* for March he proves a variety of unusual theorems that combine to provide an extremely useful technique for establishing the impossibility of certain solitaire problems. (Copies of this issue can be obtained by sending 15 cents to the National Council of Teachers of Mathematics, 1201 Sixteenth Street, NW, Washington 6, D.C.) Charosh's analysis simplifies and extends an earlier analysis by M. H. Hermary, to be found in the first volume of *Récréations Mathématiques*, edited by the French mathematician Édouard Lucas.

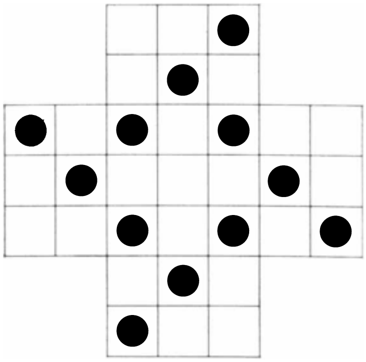
Charosh's method consists of applying a series of transformations to any starting position to see if it can be changed to the desired end position. If it can, the two positions are said to be



WALL

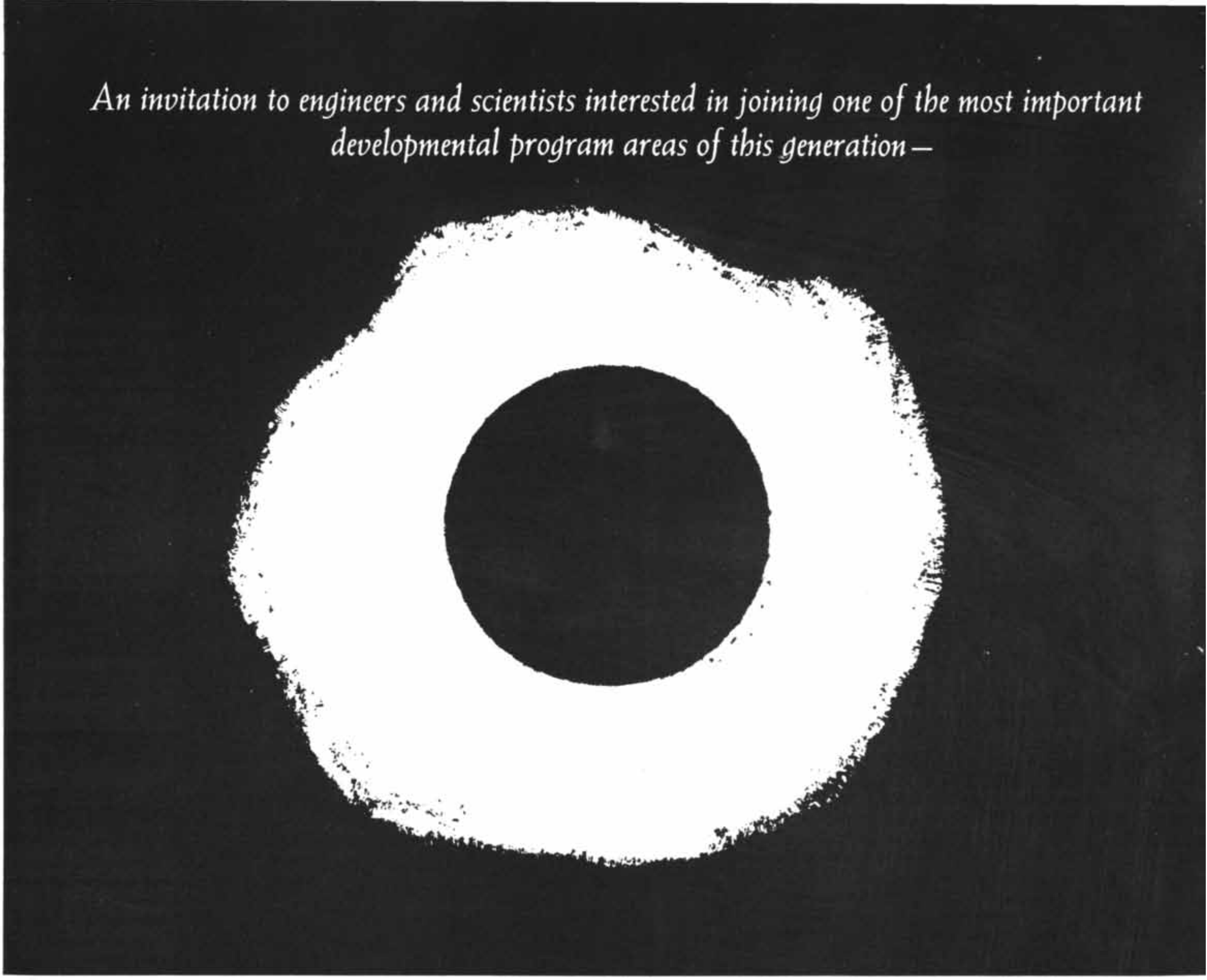


SQUARE



PINWHEEL

Play to leave these patterns on the board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"equivalent." If two positions are *not* equivalent, it is impossible to change one to the other by jumping pegs (or, alternatively, by working backward as Leibniz suggested). If two positions *are* equivalent, the problem may or may not be solvable by the rules of solitaire. In other words, the method gives to any solitaire problem, on any type of board, a necessary but not a sufficient condition of possibility.

Charosh's transformations involve any set of three adjacent cells that are in a straight horizontal or vertical line. Where there are counters on these three cells, remove them; where there are vacancies, put counters. Thus if all three cells are filled, all three counters can be removed. If all three are vacant, all three can be filled. If there are two counters, the two can be removed and a single counter can be placed on the previously empty cell. If there is only one counter, it can be removed and counters can be placed on the two previously empty cells.

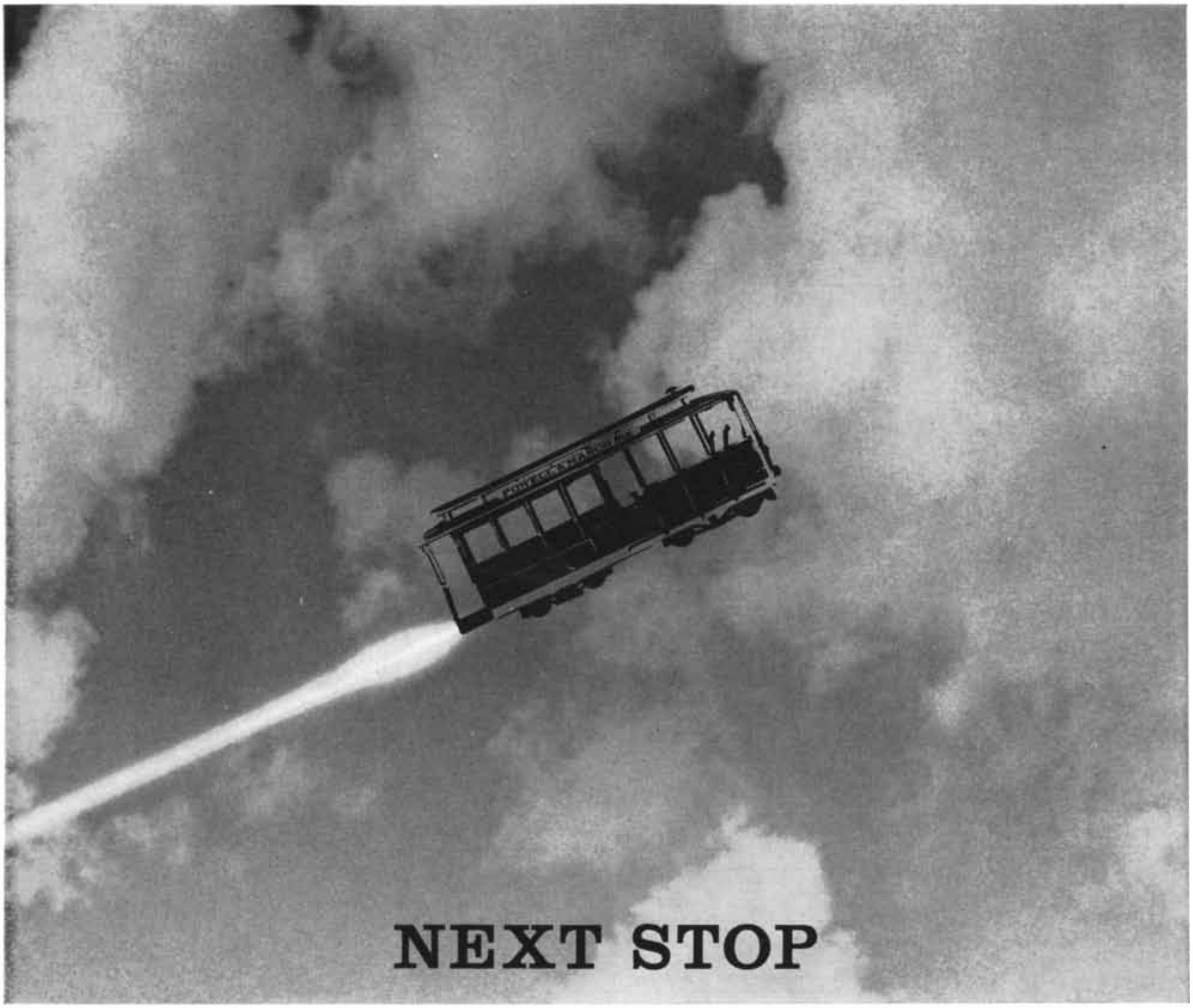
Let us apply this method to the classic problem that begins with a vacancy in the center. It can be seen at once that sets of three counters in a row can be removed until only two counters remain on, say, cells 45 and 43. Since these are the ends of the triplet 43, 44, 45, we can remove the two counters and substitute a counter on 44. We have thereby shown that the full board, with an empty cell at 44, is equivalent to an empty board with a single counter on 44; therefore the problem is not impossible. (We already know, of course, that it can be solved.) In similar fashion it is easy to see that if the game begins with a vacancy anywhere on the board, the position can be transformed by Charosh's method to a single counter on the same cell. Again, this can always be done in actual play.

Is it possible to begin with a center vacancy and end with the last counter on 45? No, it is not. There is no way that Charosh's method can be used to transform the board to a lone counter on 45. To prove this we do not have to start with a full board. We can begin with the single counter on 44 (which we know to be a possible ending) and determine how this position can be transformed to other positions with a lone counter. Thus: The counter on 44 can be removed and counters placed on 54 and 64 (because 44, 54, 64 form a triplet). The counters on 54 and 64 can in turn be taken away and replaced by a counter on 74. So a lone counter on 44 is "equivalent" to a lone counter on 74. We can put it this way: A single counter is equivalent to a single counter on any cell that can be reached by jumping over two cells in a straight line in any orthogonal direction. It is easy to see that 44 is equivalent only to cells 14, 47, 74, 41. These are the only cells on which it is possible to end a game that begins with a vacancy in the center. Practice bears this out. Any final jump that puts a counter in the center can be made in the opposite direction to put a counter in an equivalent cell. All five cells, therefore, can be reached in actual play—but no others.

Application of Charosh's method will reduce any position either to a single counter, two counters diagonally adjacent or no counters. The last cannot, of course, be reached in actual play; instead the game must end on a position equivalent to no pieces, such as three adjacent counters in a row, or two in a row with two spaces between them. It is not hard to show that any position is equivalent (transformable by Charosh's method) to its "inverse"; that is, to the same position with vacancies replaced by counters and counters by vacancies. For example, if counters are removed from two diagonally adjacent cells, say 37 and 46, the position is equivalent to an empty board with counters on those same two cells. Because there is no way to transform those two counters to a single counter, we know that it is not possible to start with vacancies at 37 and 46 and reduce the board to a single counter.

For anyone wishing to devise a new solitaire problem, Charosh's system can save endless hours of time spent in seeking solutions for impossible problems. Of course, once a problem is shown to be not impossible, the task of finding a solution remains. Sometimes a solution exists, sometimes it does not. In seeking a solution, Leibniz' method of working backward has one enormous advantage: using numbered counters and taking them in order makes it unnecessary to keep a record of each attempt. If the attempt succeeds, the numbers make it easy to reconstruct the sequence of the play.

In 1960 Noble D. Carlson, an engineer in Willoughby, Ohio, raised an interesting question: What is the smallest *square* solitaire board on which it is possible to start with a full board, except for a vacancy at one corner, and reduce the position to a single counter? Charosh's technique quickly shows that this is impossible on all squares except those with sides that are multiples of three. The



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3 × 3 square, however, proves to be unsolvable. This leaves the 6 × 6 as the most likely candidate. The solution, if there is one, will end on the corner cell left open at the start or on one of the three cells "equivalent" to it. (Let the vacancy be at cell 1, in the upper left-hand corner, and number the cells left to right. The three equivalent cells are 4, 19 and 22.)

Can it be done? Yes. Carlson himself found a 29-move solution ending on cell 22. What is wanted now is a solution beginning with a vacancy at 1 and ending at 1. If readers will send in their best solutions (assuming there *is* a 1-to-1 solution), I will comment on the results three months from now in the September issue (although I will not undertake to acknowledge each letter). "Best" may be taken to mean two different things: (1) a minimum number of moves or (2) an elegant solution in terms of some type of symmetry play.

Last month's problems are answered as follows:

1. The number 337–31770 upside down spells "Ollie Lee."

2. Hold the sum to a mirror.

3. Turn the picture upside down, circle three 6's and three 1's to make a total of 21.

4. The basket has nine white eggs and nine brown eggs. When the sum, 18, is inverted, it becomes 81, the product. Had it not been specified that the basket contained more than six eggs, three white and three brown would have been another answer. Maxey Brooke, of Sweeny, Texas, has proved that no other answers are possible.

So many readers misunderstood the six-match problem given in February that I add here a clarification. Although matches are used to form patterns, the patterns are treated as line networks. Two networks are topologically identical if one can be changed to the other by twisting or stretching. A triangle is equivalent to a square or pentagon, a line of three matches is the same as a line of two matches, and so on. With this understanding, no more than 19 topologically distinct networks can be formed on the plane with six matches. In three dimensions, one more figure (the skeleton of a tetrahedron) can be added. Seven matches on the plane will form 39 different networks. No formula is known for the number of topologically distinct networks that can be formed on the plane or in space with *n* matches.

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