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MATHEMATICAL GAMES

*About two new and two old
mathematical board games*

by Martin Gardner

The past six years have seen a remarkable upsurge of interest in mathematical board games. More people than ever before are playing the traditional games such as chess and experimenting with the new games that keep turning up in the stores. More mathematicians are analyzing the strategies of such games and more computers are being programmed to play them. This month we examine four excellent but little-known board games, two new and two old. Their playing fields can easily be drawn on paper or cardboard, the rules of play are quite simple and everyone in the family will find the contests great fun.

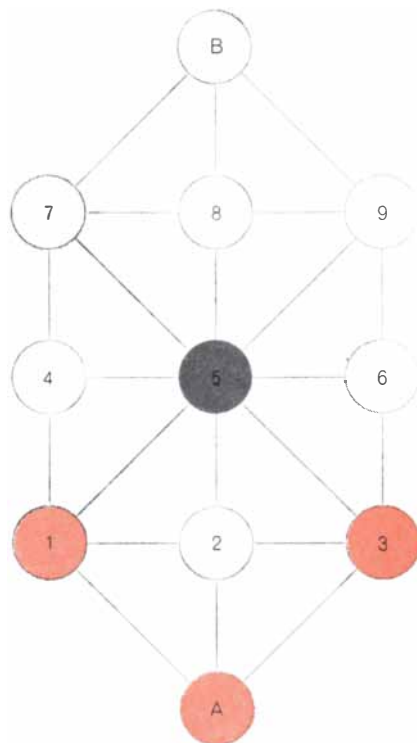
The Military Game, as it is called in France, is a splendid example of a two-player game that combines extreme simplicity with extraordinary strategic subtlety. According to Édouard Lucas, who describes the game in Volume III (pages 105–116) of his celebrated *Récréations Mathématiques*, the game was popular in French military circles during and after the Franco-Prussian War of 1870–1871. It is a pity that it has since been so completely forgotten; not one of the standard histories of board games even mentions it.

The board for the Military Game is shown in the illustration at the right with the positions labeled to facilitate description. One player—we will call him White—has three men that are initially placed on the colored spots A, 1 and 3. Black, his opponent, has only one man, which he places on spot 5 in the center. (Chess pawns can be used for men, or three pennies and a nickel.) White moves first and the game proceeds with alternate turns. Black may move in any direction along a line from one spot to a neighboring spot. White moves similarly, but only left, right or forward (straight ahead or diagonally), never backward. There are no captures. White wins if he can pin Black's piece so that it

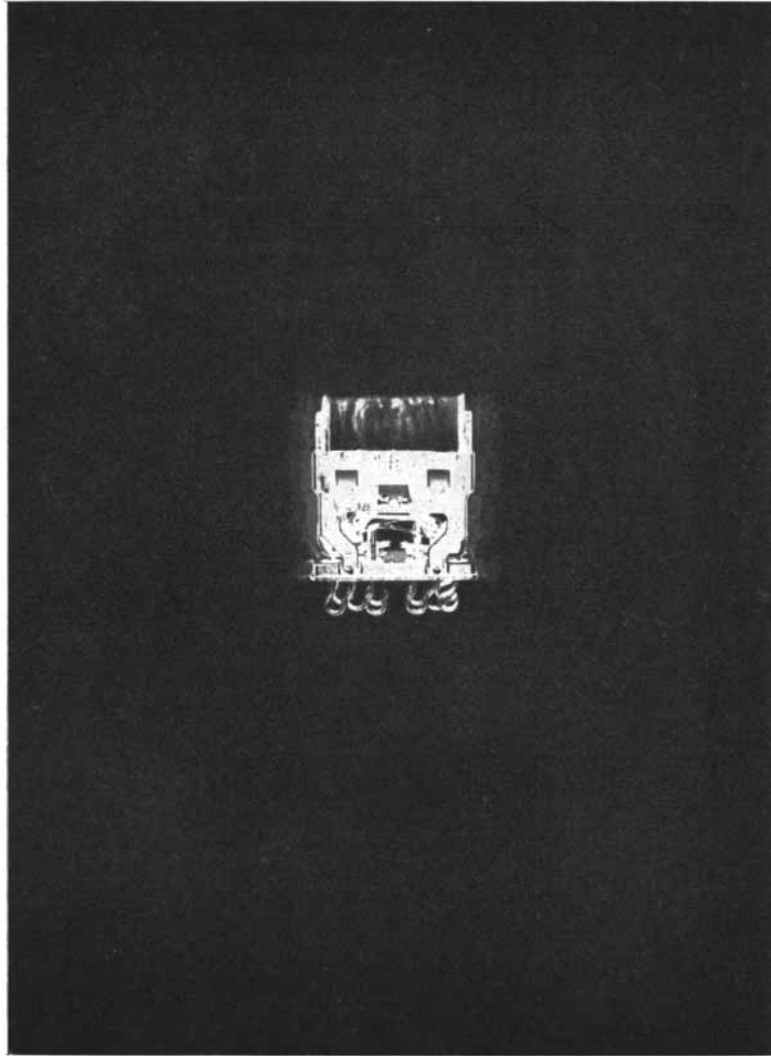
cannot move. This usually occurs with Black on spot B, but it can also occur with Black on spot 4 or 6. Any other outcome is a win for Black. He wins if he slips behind "enemy lines," making it impossible for White to pin him, or if a situation develops in which the same moves are endlessly repeated.

The game is as simple to learn as ticktacktoe, but it is more exciting to play and more difficult to analyze. Lucas is able to show that White, if he plays rationally, can always win, but there is no simple strategy and the game abounds in traps and surprises. Often the best move is the move that seems to be the worst. An experienced Black has little difficulty escaping from an inexperienced White.

Suppose we increase Black's freedom by permitting him to place his piece, at the start of the game, on *any* spot he chooses? Who now wins if both sides



The French Military Game



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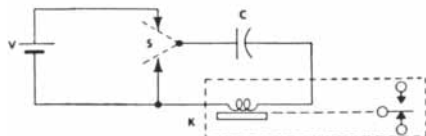
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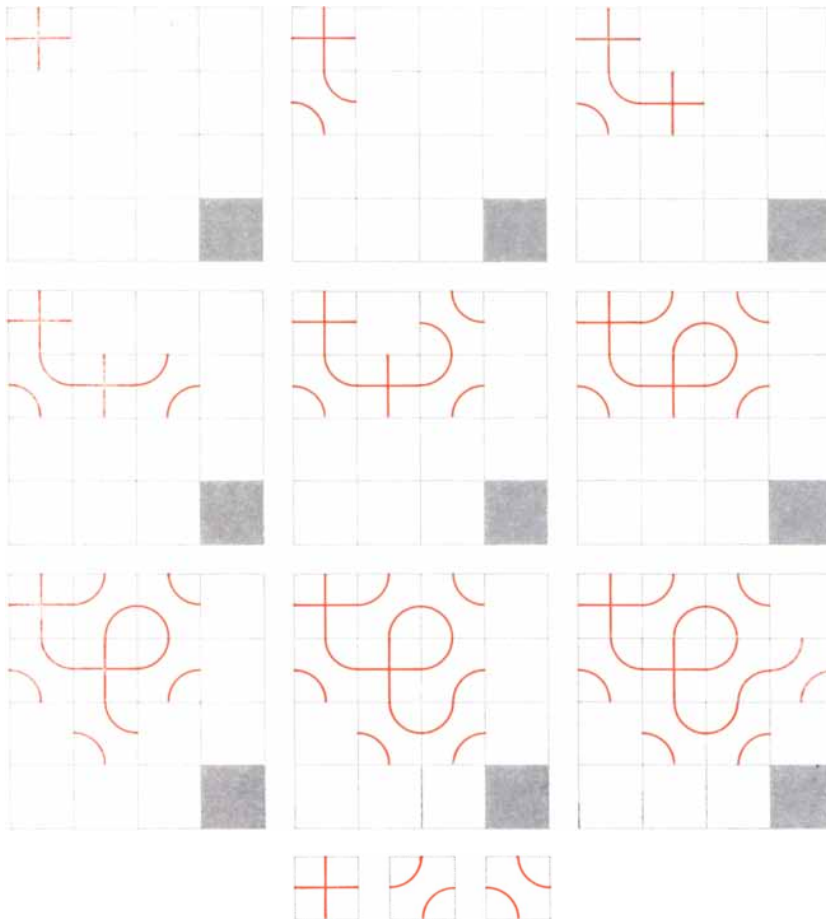
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William L. Black's game

play rationally? The answer will be given in this department next month.

Topological board games, on which players construct paths that twist about over the field, are recent developments. Hex, Bridg-it, Zig-Zag, Roadblock, Pathfinder, Squirt: these are trade names of some of the games of this type that have been marketed during the past 20 years. In 1960 William L. Black, then an undergraduate at the Massachusetts Institute of Technology (he is now working at M.I.T. for his doctorate in electrical engineering) made a study of Hex and Bridg-it, two games respectively discussed here in July, 1957, and July, 1961. An outcome of this study was a novel topological game his friends call Black.

Although marked tiles can be used, Black is easily played as a pencil-and-paper game on a checkered field. The size of the field is optional; the standard eight-by-eight field seems ideal, but it is simpler to explain the game on the smaller four-by-four. After the field is drawn the first player starts the game by making a cross in the upper left corner cell as shown in the first drawing in the

illustration above. The second player continues the path by making one of three permissible marks in a cell adjacent to the first cell marked. The three marks, shown at the bottom of the illustration, represent one of the three ways in which the path can be joined to an open side of the square; the second is added to connect the remaining two sides.

The players alternate moves. Each move must extend the path into a neighboring cell. Each player tries to avoid running the path into a border of the field. If he is forced to carry the path to the border, he loses the game. If he succeeds in extending the path into the lower right corner cell [*shown shaded*], he wins. The illustration shows successive moves of a typical short game. The first player wins by forcing his opponent to play in the upper right corner cell, where any mark will carry the path to the edge of the field. (Note that the cross extends the path only along one of its arms, although the other arm may become part of the path as the result of a later play.)

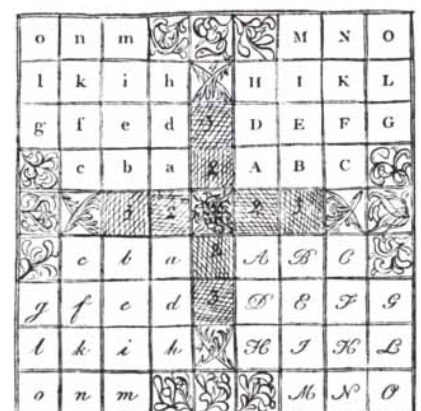
The game of Black is of special in-

terest because soon after it was conceived a friend of Black's, Elwyn R. Berlekamp (who is also currently pursuing his Ph.D. in electrical engineering at M.I.T.), hit on an elegant strategy that guarantees a win for one of the players. The strategy applies to rectangular fields of any size or shape. Since knowledge of the strategy destroys all interest in actual play, I temporarily refrain from giving it. Readers are urged to play the game and see if they can match Berlekamp's brilliant insight before it is disclosed next month.

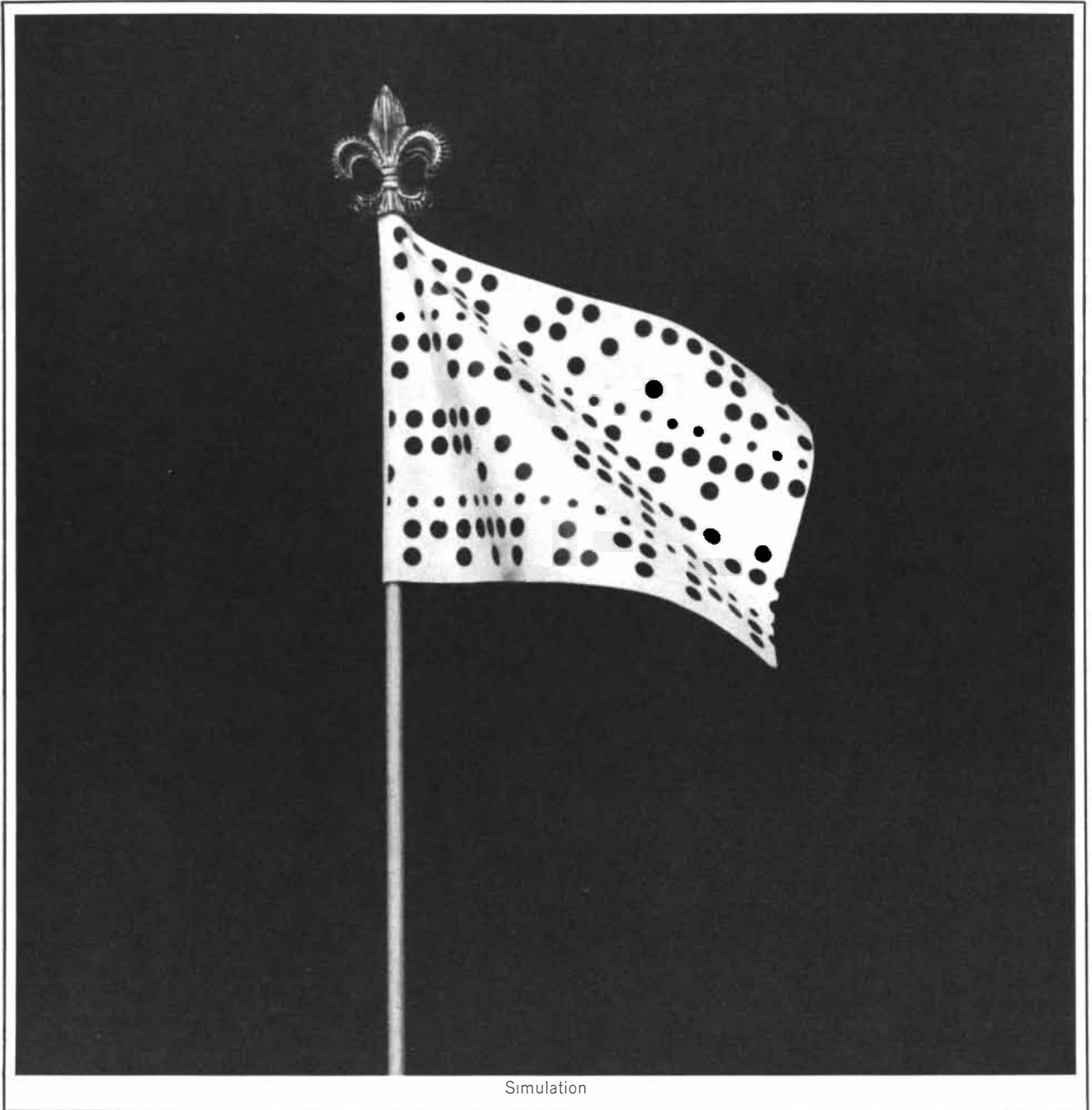
One of the best of many medieval board games is a game that seems to have been first played in Scandinavian countries as early as the fourth and fifth centuries, when it was called *tafl*. In later centuries it was known as *hnefatafl*. The Norsemen introduced the game to Britain, where it was the only board game played by the early Saxons until it began to be replaced by chess in the 11th and 12th centuries. H. J. R. Murray, in his *History of Board-Games Other than Chess*, gives reasons for thinking that this is essentially the same game that was still being played in the 16th century in Wales, under the name of *taulbwrd*, and in the 18th century in Lapland, where it was known as *tablut*.

It was Murray who discovered that the great Swedish botanist Carolus Linnaeus included a full description of *tablut* in an extensive diary he kept during his exploration of Lapland in 1732. An English translation of the diary, by Sir James Edward Smith, was published in London in 1811 with the title *Lachesis Lapponica: or a Tour of Lapland*. The illustration below is a reproduction of the *tablut* board as it is shown on page 55 of Volume II of this edition.

White pieces, representing light-haired Swedes, include a single king and eight warriors. Black pieces, 16 in



The game of tablut



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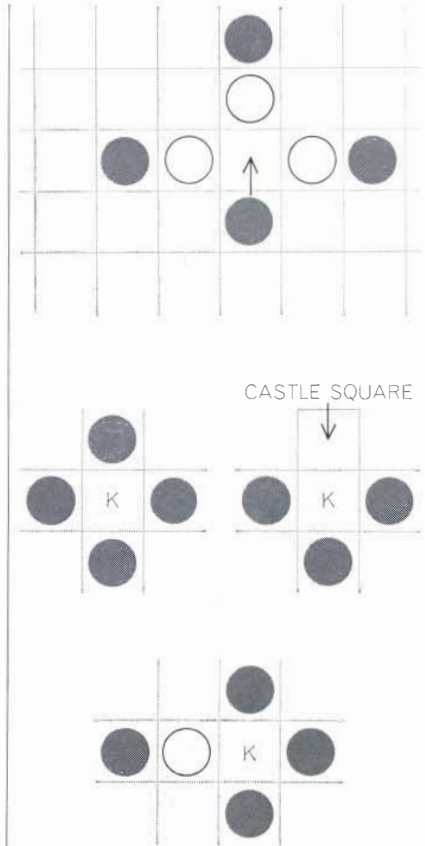
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Methods of capture in tablut

number, represent Muscovite warriors. (It is convenient to use a white chess king and eight white pawns for the Swedes. Black chessmen can be used for the Muscovites, but all must be regarded as identical pieces.) Each black and white piece, including the king, moves like a rook in chess, that is, an unlimited distance along vacant cells in a straight line paralleling a side of the board.

The game begins with the Swedish king occupying the center square, which is known as the castle. Only the king is permitted to stand in the castle, although any piece may move through it when it is vacant. Surrounding the king, on the eight shaded squares, are his eight warriors. The Muscovites occupy the 16 decorated squares at the four sides of the board.

Either player may open the game. Enemy pieces are captured by a pincer move that consists of occupying adjacent cells on opposite sides of a piece, the three pieces being in the same row or column. For example, if Black makes the indicated move, he captures the three white pieces simultaneously [see top drawing in illustration above]. If a piece moves between two enemy pieces, how-

ever, it is not captured by them. The king may take part in capturing enemy pieces, but he himself is captured only if he is surrounded on all four sides by four enemy pieces or by three enemy pieces and the castle square [*middle drawing*]; he cannot move from his castle into such a formation without being captured.

Linnaeus adds that when the king is in his castle, with three enemy warriors on three sides and one of his own men on the fourth side, the Swedish warrior is taken if a Muscovite moves to the cell next to the Swede on the side opposite the king [*bottom drawing*].

Black's objective is to capture the king. If this occurs, the Muscovites win. White's objective is to allow the king to flee the country by reaching any cell on the perimeter of the board. Whenever there is an unobstructed path along a row or column by which the king can reach the border, White must warn Black by saying "Raichi!" (a remark similar in function to "Check!" in chess). If there are two escape paths, White calls out "Tuichu!" Of course "Tuichu!" announces a win for White because there is no way Black can block two escape routes with a single move.

Sidney Sackson, a New York City engineer who makes a hobby of collecting board games (he owns about 500 actual boards and his files contain details of hundreds of others), knows of only one occasion on which *tablut* has been made and sold in this country. In 1863 it was issued as a Civil War game called Freedom's Contest, or the Battle for the Union. This game is identical with *tablut* except that the king is called the "Rebel chief" and the pieces are Rebel and Union soldiers. The Rebel chief is limited to a maximum move of four spaces. The traditional game seems to favor White, so perhaps this restriction was intended to redress the balance.

Sackson is himself the inventor of many unusual board games, one of the best of which he calls Focus. It is played with 36 counters, half of them one color and half another. Small poker chips of the interlocking variety make excellent pieces. They are placed initially on an eight-by-eight board from which three cells at each corner have been removed. The illustration at the bottom left on the next page shows how the pieces (black and colored in this case) are arranged.

Either side may move first. A move consists of moving a "pile" of pieces (at the outset all piles are one chip high) as many spaces as there are pieces in the pile. Moves are vertical or horizontal, never diagonal. The four pos-



Photographic interpretation by J. Frederick Laval

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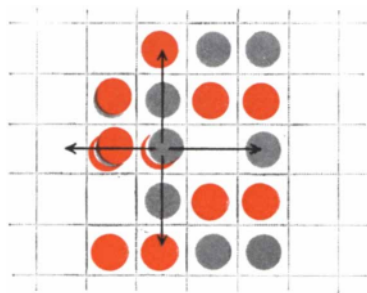


All qualified applicants will receive consideration for employment without regard to race, creed, color or national origin. U.S. citizenship required.

sible moves of one colored piece at the start of a game are shown in the illustration below. If the piece moves up, it lands on a vacant square. A move to the right puts it on top of another colored piece, to the left or down puts it on top of a black piece. The last three moves form two-high piles. Such piles may be moved two spaces in any direction. Piles of three, four and five pieces move three, four and five spaces respectively. A pile is controlled by the player who owns the piece on top. In moving it does not matter whether the intervening cells are empty or occupied by piles controlled by either player. Passed-over pieces are not affected in any way. A move may end on a vacant cell or on another pile. The illustration at the top of the page shows the possible moves of a two-high pile.

Piles may not contain more than five pieces. If a move produces a pile of more than five, all pieces in excess of five are taken from the bottom of the stack. If they are enemy pieces, they are considered captured and are removed from the game. If they belong to the player making the move, they are placed aside as reserves. At any time during the game a player may, if he wishes, take one of his reserve pieces and place it on any cell of the board, empty or otherwise. It has the same effect as a moved piece: if it goes on a pile, the pile belongs to the player who placed it. Using a reserve piece substitutes for a move on the board.

A player may, if he wishes, make a move of fewer spaces than the number of pieces in the pile being moved. He does this by taking from the *top* of the pile as many pieces as the number of spaces he wishes to move. The rest of the pieces stay where they are. For example, a player may take the top three pieces of a five-high pile and move them three



Moves in the game of Focus

spaces. The pile that *remains* after such a move belongs to the player who owns the piece on top.

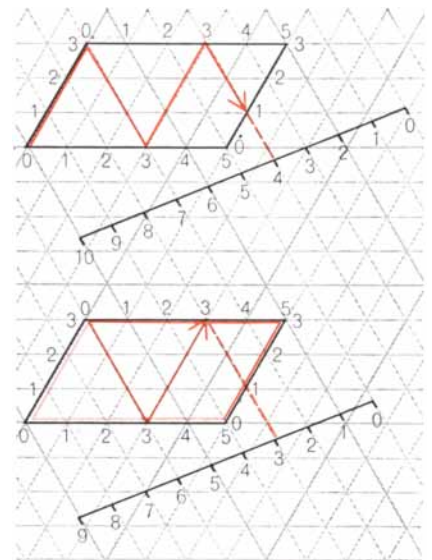
When a player is unable to move (that is, controls no piles and has no reserves), the game is over and his opponent wins. Readers who find this game stimulating may wish to send 25 cents to Sidney Sackson, 1287 Arnow Avenue, Bronx 69, N.Y., for his copyrighted booklet giving the rules in more detail, including a four-handed game and a number of valuable hints for skillful play.

Number play and football are blended cleverly in a game called Number Football, recently developed by the Benedictine Fathers at Benet Lake, Wis. For \$1 the Fathers will send postpaid a 24-page booklet explaining the game, together with a pad of gridiron blanks for scoring. And I must not fail to mention that a hard-cover book by Robert Abbott—whom readers of this department know as the inventor of Eleusis, the induction card game—will be published this month by Stein and Day. Although titled *Abbott's New Card Games*, it includes a remarkable mathematical board game of the chess variety. In addition to four card games reprinted from Abbott's earlier paperback, it contains four excellent new card games unlike any the reader has ever played before.

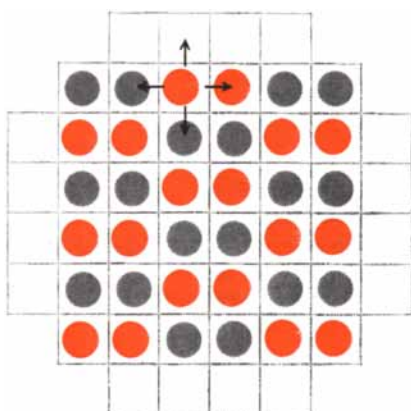
Last month's problem was: Given a 10-gallon keg filled with beer and two vessels of three-gallon and five-gallon capacity, how can one (in the minimum number of operations) drink a quantity of beer and leave equal amounts in each of the three vessels? Since the vessels measure only integral amounts, the beer to be divided into thirds must be a multiple of three: three, six or nine gallons. The first two amounts can be eliminated because in both cases a third of the amount is less than the capacity of each vessel. (After any pouring operation at least one vessel must be either empty or full. Neither situation would obtain if each vessel contained less than its capacity.) We conclude, therefore,

that one gallon must be drunk, leaving nine to be divided into thirds.

The ball-bouncing computer explained last month traces a minimum path that measures one gallon [see upper graph in illustration below]. After the gallon (in the three-gallon vessel) is drunk, four gallons remain in the 10-gallon keg, five in the five-gallon vessel. The three-gallon vessel is empty. This new situation is diagrammed as shown in the lower graph. The ball must now reach a point that marks three gallons in each container. The minimum path is shown in color, with two alternative steps in a lighter shade. Counting the drinking of the gallon as an "operation," the complete solution involves nine operations, which are shown below the two graphs.



Solution to last month's problem



Sidney Sackson's game of Focus