# MATHEMATICAL GAMES

## *A new pencil-and-paper game based on inductive reasoning*

by Martin Gardner

Many games and pastimes have flimsy analogies with induction, that strange procedure by which scientists observe that some ostriches have long necks and conclude that all unobserved ostriches also have long necks. In poker and bridge, for instance, players use observational clues to frame probable hypotheses about an opponent's hand. A cryptographer guesses that a certain "pattern word," say BRBQFBQF, is NONSENSE, then tests this inductive conjecture by trying the letters elsewhere in the message. An old parlor entertainment involves passing a pair of scissors around and around a circle of players. As each person transfers the scissors he says "Crossed" or "Uncrossed." Those acquainted with the secret rule tell a player when he says the wrong word, and the joke continues until everyone has guessed the rule inductively. The scissors' blades are a red herring; a player should say "Crossed" if and only if his *legs* are crossed.

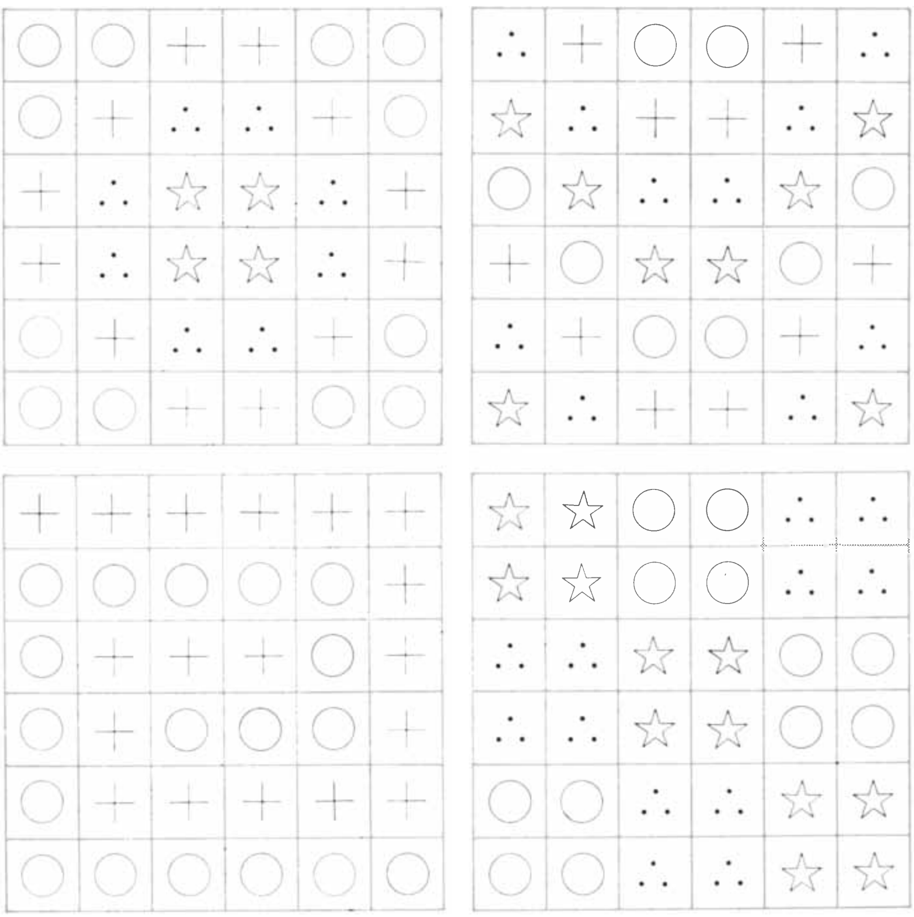
Familiar games such as Battleship and Jotto have slightly stronger analogies with scientific method, but the first full-fledged induction game was Eleusis, a card game invented by Robert Abbott and first explained in this department in June, 1959. (Fuller details are in *Abbott's New Card Games,* a Stein and Day hard-cover book in 1963 and a Funk & Wagnalls paperback in 1969.) Eleusis intrigued many mathematicians—notably Martin D. Kruskal of Princeton University, who worked out an excellent variant that he described in 1962 in a privately issued booklet, *Delphi—a Game of Inductive Reasoning.*

In Eleusis and Delphi a secret rule, specifying the order in which single cards may be played, corresponds to a law of nature. Players try to guess the rule inductively and then (like scientists) test their conjectures. This month I shall explain a new type of induction game called Patterns, devised by Sidney Sackson and included in his delightful book *A Gamut of Games,* to be published this fall by Random House.

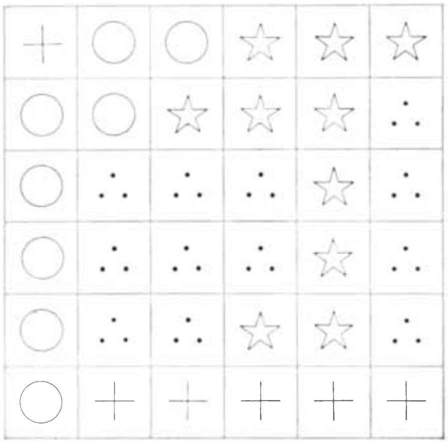
Patterns is a pencil-and-paper game that can be played by any number of people, although preferably no more than six. It differs markedly from Eleusis and Delphi, but it shares with them such a striking similarity to scientific method that many thorny problems about induction that have needled philosophers of science ever since David Hume showed induction has no logical justification have pleasant analogues in the game.

Each player draws a square six-by-six grid on a sheet of paper. A player called the Designer (the role of Designer passes to another player with each new game) secretly fills in his 36 cells by drawing in each cell one of four different symbols. Sackson suggests the four shown in the illustration at the left, but any other four may be used. The Designer, who can be regarded as Nature, the Universe or the Deity, is free to mark the cells as he likes; they may form a strong or a weakly ordered pattern, a partially ordered pattern or no pattern at all. However (and here Sackson adopts the brilliant original idea of Abbott's), the method of scoring is such as to impel the Designer to create a pattern, or a regularity of nature, that is easy to discover for at least one player and yet difficult enough to be missed by at least one other player.

Four typical patterns given in Sackson's book are arranged roughly in order of difficulty [*see illustration at left*]. All have some type of visual symmetry, but nonsymmetrical forms of order can be used if the players are mathematically sophisticated. For example, a Designer might take the cells in sequence, left to right and top to bottom, putting a plus sign in each cell whose number is prime and a star in all the remaining cells. The basis for ordering the Master Pattern is intimately bound up with the Designer's estimate of the abilities of the other players because, as we shall see, he makes his



*Patterns for Sidney Sackson's induction game, all showing forms of symmetry*



*How is this pattern ordered?*

highest score when one player does very well and another very poorly. Can the reader discern the simple basis for the nonsymmetrical ordering shown above before it is given next month?

The Designer puts his sheet face down on the table. Any player may now make inquiries by drawing on his own grid a small slant line in the lower left-hand corner of any cell about which he seeks information. His sheet is passed face down to the Designer, who must enter the correct symbol in each cell in question. There are no turns. A player may ask for information whenever he wants, and there is no limit to the number of cells about which he may inquire. Each request represents an observation of nature—or an experiment, which is simply a controlled way of making special observations; cells filled in by the Designer correspond to the results of such observations. A player could ask for information about all 36 cells and obtain the entire pattern at once, but this is not to his advantage because, as we shall learn, it would give him a score of zero.

When a player believes he has guessed the Master Pattern, he draws symbols in all his untested cells. To make it easy to identify these inductions, guessed symbols are enclosed in parentheses. If a player decides he cannot guess the pattern, he may drop out of the game with a zero score. This is sometimes advisable because it prevents him from making a minus score and also because it inflicts a penalty on the Designer.

After all players have either filled in all 36 cells or dropped out of the game, the Designer turns his Master Pattern face up. Each player checks his guesses against the Master Pattern, scoring +1 for every correct symbol, −1 for every incorrect symbol. The sum is his final score. If he made a small number of inquiries and correctly guessed all or most of the entire pattern, his score will be high. If he has more wrong than right guesses, his score is negative. High scorers are the brilliant (or sometimes lucky) scientists; poor scorers are the mediocre, impulsive (or sometimes unlucky) scientists who rush poorly confirmed theories into print. Dropouts correspond to the mediocre or overcautious scientists who prefer not to risk framing any conjecture at all.

The Designer's score is twice the difference between the best and the worst scores of the others. His score is reduced if there are dropouts. Five points are subtracted for one dropout, 10 for each additional dropout. Sackson gives the following examples of games with a Designer (*D*) and players *A, B, C:*

If *A* scores 18, *B* scores 15 and *C* scores 14, *D*'s score is 8, or twice the difference between 18 and 14.

If *A* scores 18, *B* scores 15 and *C* scores −2, *D*'s score is 40, or twice the difference between 18 and −2.

If *A* scores 12, *B* scores 7 and *C* drops out with a score of 0, *D*'s score is 19, or twice the difference between 12 and 0, with five points deducted for the single dropout.

If *A* scores 12 and *B* and *C* both give up, *D* scores 9. This is twice the difference between 12 and 0, with five points deducted for the first dropout, 10 for the second.

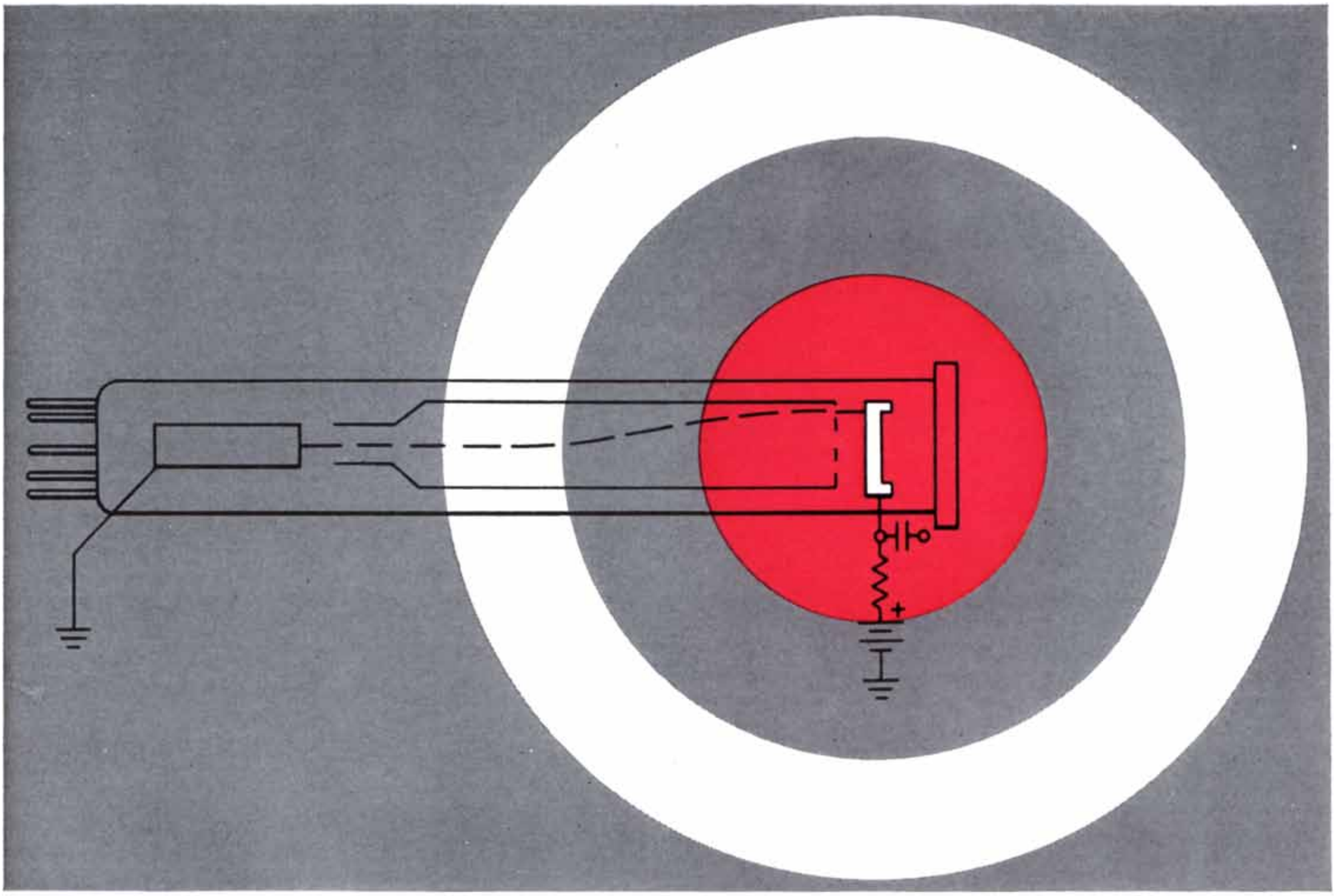
If all three players drop out, *D*'s score is −25. His basic score is 0, with 25 points subtracted for the three dropouts.

An actual game played by Sackson suggests how a good player reasons [*see illustration below*]. The five initial inquiries probe the grid for evidence of symmetry [*left*]. The sheet is returned with the five symbols filled in [*middle*]. A series of additional inquiries brings more information [*right*]. It looks as if the pattern is symmetrical around the diagonal axis from top left to bottom right. Since no stars have appeared, Sackson induces that they are absent from the pattern.

Now comes that crucial moment, so little understood, for the intuitive hunch or the enlightened guess, the step that symbolizes the framing of a hypothesis by an informed, creative scientist. Sackson guesses that the top left-hand corner cell contains a circle, that the three cells flanking it all have plus marks and that, continuing down the diagonal, the pluses are flanked by three-spot symbols, the pattern repeating itself with larger borders of the same three symbols in the same order. To test this conjecture with as few new inquiries as possible Sackson asks for information on only two more cells, the two cells shown empty but with slant lines on the grid at the right in the illustration below.

If those cells do not contain circles, his conjecture is false. As the philosopher Karl Popper maintains, the "strongest"

*Three stages in probing for the Master Pattern*



**A solid state target in a camera tube**

Bell System PICTUREPHONE® service will need small, reliable TV camera tubes for use in offices and homes, where lighting ranges from dim to very bright. Conventional vidicon tubes are unsuitable, so Bell Labs developed a new kind.

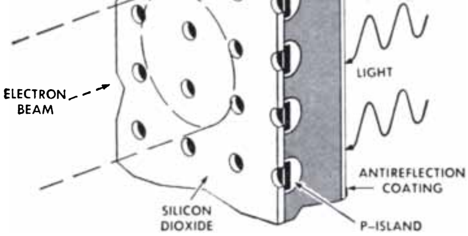
The heart of the new tube is a light-sensitive target containing nearly 700,000 silicon photodiodes in an area less than a half inch square. They are made by diffusing boron, a p-type impurity, through a silicon-dioxide mask into n-type silicon.

A scanning electron beam charges the p material negatively, reverse-biasing the diodes. Holes, created by incident light, are collected by the electric field at the p-n junctions, and individual diodes discharge by an amount proportional to the local light intensity. Recharging of the diodes by the scanning electron beam produces a varying current ... the output signal.

Among the tube's advantages:

Its target tolerates high-temperature baking ... a processing step to improve reliability. Conventional vidicon targets cannot stand this.

Silicon's high thermal conductivity and chemical stability help make the new tube immune to "burn-in" (degradation of performance from continuous exposure to a fixed image, very bright light, or a strong scanning electron beam).




The time between a change in target illumination and a like change in output is much shorter with the new target. This improves response to fast-changing scenes.

The light-sensitive face of the new target is optically flat. So, a multilayer antireflection coating can be applied for better sensitivity and minimum received-picture "halo".

Silicon targets have relatively uniform response through the visible and near-infrared—from 4,000 to 9,000 A. Quantum efficiency (electrons per photon) exceeds 0.5. So, these targets have at least 10 times the sensitivity of a standard vidicon camera tube in incandescent light.

This new camera tube is in the latest model PICTUREPHONE set, now undergoing field trials.

**From the Research and Development Unit of the Bell System—**

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conjecture is the one that is easiest to falsify, and Popper considers this the equivalent of the "simplest" conjecture. In Sackson's game the strongest (and simplest) conjecture is that every cell contains the same symbol, say a star. It is strong because a single inquiry about *any* cell, answered by anything but a star, falsifies it. The weakest conjecture is that each cell contains one of the four symbols. Such a hypothesis can be completely confirmed. Since no inquiry can falsify it, however, it is a true but useless hypothesis, empty of all empirical content because it tells one nothing about the Master Pattern.

The circles turn out to be where Sackson expected them. This increases what the philosopher Rudolf Carnap calls the "degree of confirmation" of Sackson's hypothesis in relation to the total evidence he has bearing on it. Sackson decides to take the inductive plunge and "publish" his conjecture. He fills in the empty cells of his grid. When his pattern is compared with the Master Pattern [*see illustration below*], a count of the guessed symbols (in parentheses) shows that Sackson has 20 right and one wrong, for a score of 19.

The single star Sackson missed is unexpected, but it is typical of the surprises Nature often springs. Science is a complicated game in which the universe seems to possess an uncanny kind of order, an order that it is possible for humans to discover in part, but not easily. The more one studies the history of the game of science, the more one has the eerie feeling that the universe is trying to maximize its score. A splendid recent example is the independent discovery by Murray Gell-Mann and Yuval Ne'eman of the "eightfold way." This is a symmetry pattern, defined by a continuous group structure, into which all the elementary particles seem to fit. As soon as enough information had accumulated the pattern was simple enough to be spotted by two physicists, and yet it remained complicated enough to be missed by all the other players.

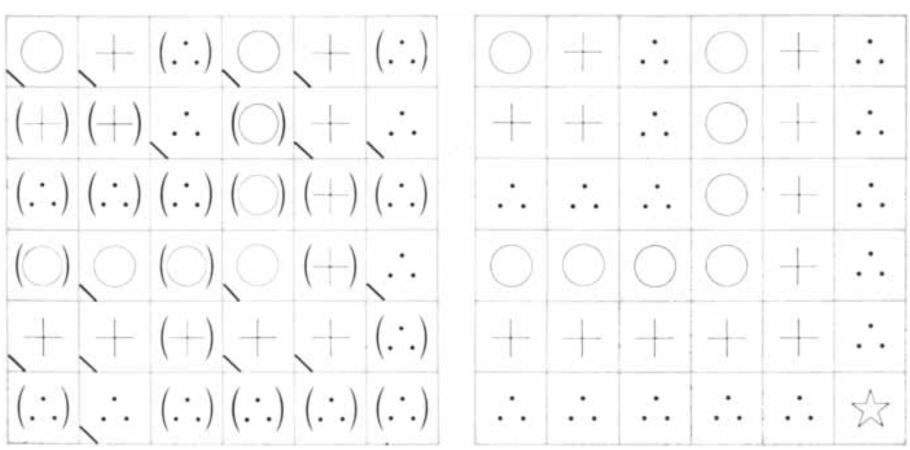
Sackson, the inventor of Patterns, is a professional engineer who works on steel bridges and buildings. Collecting, studying and inventing games has been his lifelong avocation. He owns what is probably the largest private collection of modern proprietary games, books on games, and notes obtained by painstaking research in the world's great libraries and museums. He has invented hundreds of games. The first, he discloses in his book, was invented when he was in the first grade; it had to do with circling words on a page and joining them in chains. The first board game he owned was Uncle Wiggily, a track game that is still on the market. He immediately modified it by altering its rules and substituting toy soldiers for rabbits to make it a war game.

All Sackson's marketed games emphasize intellectual skill rather than luck. Acquire, a game based on the theme of investing in hotel chains, has been his best-selling item. His other commercial games include The Case of the Elusive Assassin (a logic game based on Venn diagrams), Focus, Bazaar and two card games using special decks, Venture and Monad, both issued this year. Tam-Bit, Take Five, Odd or Even, Tempo and Interplay are among other Sackson board games that will go on sale in 1970.

*A Gamut of Games* is unique in that almost every one of its 38 games will be completely unfamiliar to any reader. All can be played with equipment that is easily acquired or constructed: cards, dice, dominoes, counters and checkerboards. Twenty-two are Sackson originals. The others are either creations of Sackson's game-inventor friends or old, forgotten games that deserve revival. No two readers will, of course, have the same reaction to every game. I particularly like Knight Chase, played on a chessboard with one black and one white knight and 30 small counters. It is the invention of Alexander Randolph, Czechoslovakian-born but now living in Japan, who has several excellent games on the U.S. market: Oh-Wah-Ree (based on the African game of mancala), Twixt and Breakthru. Another mathematically appealing game (which Sackson found in an 1890 book) is Plank, a version of ticktacktoe played with 12 tricolored cardboard strips. The reader will find a valuable bonus in the book's final section: brief reviews of more than 200 of the best adult games now on sale in this country.

Sackson's informal text is interspersed with personal anecdotes and snippets of surprising historical data. Until I read his book I did not know that the 17th-century poet Sir John Suckling invented cribbage, or that Monopoly, the most successful of all proprietary board games, is derived from The Landlord's Game, which was patented in 1904 by one Lizzie J. Magie and was intended to teach Henry George's single-tax theory. Sackson reproduces the patent drawing of the Magie board; the similarity to Monopoly is obvious.

Marketed board games, Sackson points out, tend to reflect major events and interests of the time. Although he does not mention it, an amusing example of this is The Money Game, a card game invented by Sir Norman Angell, who received the Nobel peace prize for 1933. The special cards and miniature money for this stock-market-speculation game were packaged with a 204-page explanatory book issued by E. P. Dutton, with puffs on the jacket by Walter Lippmann, John Dewey and noted economists. (A book by "Adam Smith" with the same title and topic has recently been a best seller.) Why is Angell's Money Game so grimly amusing? The year of its publication was 1929.



*Player's grid* (left) *compared with Master Pattern* (right)

Last month's questions are answered as follows:

1. The anagram of "moon starers" is "astronomers."

2. The solution to SPIRO $\times$ 7 = AGNEW is 14,076 $\times$ 7 = 98,532. This unique solution was first published by E. P. Starke in *The American Mathematical Monthly*, Volume 53, January, 1946, page 45. In *Mathematics Magazine*, Volume 42, March, 1969, pages 102–103, David E. Daykin gives two tables of computer re-

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Stromberg DatagraphiX Inc., a General Dynamics subsidiary, P.O. Box 2449, San Diego, California 92112 (714) 283-1038 TWX: (910) 335-2058 sults that list the number of different solutions of $A = kB$, in all number systems with bases 3 through 15, where $k$ is any number from 2 through 14 and $A$ and $B$ are numbers that together contain all 10 digits once each or all nine digits (0 excluded) once each. Solutions in which $A$ or $B$ begin with 0 are not considered. In Dr. Matrix' problem (base 10 and all 10 digits) there are 48 solutions for $k = 2$, six for $k = 3$, eight for $k = 4$, 12 for $k = 5$, none for $k = 6$, one for $k = 7$, 16 for $k = 8$ and three for $k = 9$.

3. The earliest A.D. year with the property explained last month for 1969 is the year 19. The square of 19 is 361, the last two digits of which, upside down, are also 19. (The trivial case of year 1 was excluded.)

4. A year can have no more than four "perverse months"—months requiring six calendar rows—and no fewer than two. It can have no more than three Friday the 13th's and no fewer than one. As Kirby A. Baker has shown, the number of perverse months in a year plus the number of Friday the 13th's is four, except for years beginning on Sunday and leap years beginning on Saturday, in which cases the sum is five. The last year with four perverse months was 1944; the next will be 1972, with no more in this century.

| Year | | Leap year | |
|---|---|---|---|
| | | JAN | 1 2 3 |
| JAN | 1 2 3 | FEB | 1 |
| MAR | 1 2 3 | MAR | 1 2 3 |
| APR | 1 2 | APR | 1 2 |
| MAY | 1 2 3 | MAY | 1 2 3 |
| JUN | 1 2 | JUN | 1 2 |
| JUL | 1 2 3 | JUL | 1 2 3 |
| AUG | 1 2 3 | AUG | 1 2 3 |
| SEP | 1 2 | SEP | 1 2 |
| OCT | 1 2 3 | OCT | 1 2 3 |
| NOV | 1 2 | NOV | 1 2 |
| DEC | 1 2 3 | DEC | 1 2 3 |

***Pseudochronian year and leap year***

To prove that four and two are the maximum and minimum numbers of perverse months in any year, Baker suggests the following exercise: Eliminate the last four weeks of each month. The remaining days (two or three per month except for February, which in leap years has one day and in nonleap years has no days) form the calendar used by inhabitants of a planet that Baker calls Pseudochron. Pseudochronian years are 29 or 30 days long, but their weeks have seven days, like ours.

Pseudochronian years have two schema [*see illustration on this page*]. In each case a Pseudochronian month starts and ends on the same days as a corresponding earth month for a corresponding year. For example, 1969 on the earth started and will end on Wednesday. Label the dates on the Pseudochronian nonleap year, beginning with Wednesday, and you will find the year ends on Wednesday. Moreover, each Pseudochronian month begins and ends on the same days as the corresponding earth months of 1969.

A perverse earth month corresponds to a Pseudochronian month that includes both Saturday and Sunday. If we try the seven possible ways to label the dates of a Pseudochronian nonleap year, starting the year with each of the seven days, we find that it can have no fewer than two perverse months and no more than three. A similar procedure with the Pseudochronian leap year gives no fewer than two perverse months, but if and only if the year begins with Saturday, there are four perverse months.

Since Friday the 13th occurs only in months starting with Sunday, a similar test will prove that no year can contain fewer than one Friday the 13th or more than three. I leave it to readers to establish the relationship between perverse months and Friday the 13th's.

A proof by B. H. Brown that the 13th is more likely to fall on Friday than on any other day appeared in *The American Mathematical Monthly*, Volume 40, December, 1933, page 607; a proof by S. R. Baxter (done at the age of 13) appeared in *The Mathematical Gazette,* Volume 53, May, 1969, pages 127–129.

Many short proofs of the maximum and minimum numbers of Friday the 13th's in a year have been published. For a good recent discussion of this problem see William T. Bailey, "Friday-the-Thirteenth," in *The Mathematics Teacher*, Volume 62, May, 1969, pages 363–364.