Logic Programming
View Evaluation

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Bottom-Up Evaluation

Method

Start with dataset

Apply rules repeatedly to produce closure

Repeat up the stratum hierarchy

Evaluate query on the result

Disadvantages

Generates large numbers of irrelevant conclusions

Does not work with infinite extensions

Top-Down Evaluation

Method

Start with query to be answered

Apply rules repeatedly to reduce to subqueries

Continue until reaching data level

Match base level subgoals against dataset

Disadvantages

Slightly harder to understand

Sometimes recomputes subgoals

Susceptible to *avoidable* infinite loops

Programme

Top-Down Processing of Ground Goals and Rules

Unification

Top-Down Processing of Goals and Rules with Variables

Ground Goals and Rules

Sketch of Procedure for Ground Case

Given a query, a dataset, and a ruleset, do the following.

(1) If the predicate in the query is a **base predicate**, succeed if and only if query is in dataset.

(2) If the query is a **negation**, evaluate target and succeed if and only if fail to prove.

(3) If the query is a **conjunction**, succeed iff succeed on all conjuncts.

(4) If the predicate in the query is a **view predicate**, evaluate the body of each rule defining that predicate and succeed if and only if succeeds on at least one rule.

Example

Dataset

```
p(a)
p(b)
p(c)
q(d)
```

Ruleset

```
s(c) :- p(a) & q(b)
s(c) :- p(b) & t(c)
s(c) :- p(c) & ~q(c)
t(c) :- p(a) & p(d)
```

Example

Dataset

```
p(a)
p(b)
p(c)
q(d)
```

Ruleset

```
s(c) :- p(a) & q(b)
s(c) :- p(b) & t(c)
s(c) :- p(c) & ~q(c)
t(c) :- p(a) & p(d)
```

Top Down Evaluation

```
s(c)?

p(a) & q(b)?      p(b) & t(c)?      p(c) & ~q(c)?
X                                   Success

                  p(a) & p(d)?
                  X
```

Unification

Unification

Unification is the process of determining whether two expressions can be *unified*, i.e. made identical by appropriate substitutions for their variables.

Example: `p(a,Y)` and `p(X,b)` can be unified. If we replace `X` by `a` and `Y` by `b`, we end up with `p(a,b)` in both cases.

Substitutions

A *substitution* is a finite set of pairs of variables and terms, called *replacements*.

$$\{\mathtt{X}\leftarrow\mathtt{a},\ \mathtt{Y}\leftarrow\mathtt{f(b)},\ \mathtt{Z}\leftarrow\mathtt{V}\}$$

Domain: $\{\mathtt{X}, \mathtt{Y}, \mathtt{Z}\}$
Range: $\{\mathtt{a}, \mathtt{f(b)}, \mathtt{V}\}$

NB: Domain elements *must* be variables.
NB: Replacements *may* contain variables.

Application

The result of applying a substitution σ to an expression φ is the expression $\varphi\sigma$ obtained from φ by replacing every occurrence of every variable in the substitution by its replacement.

$$\texttt{q(X,Y)}\{\texttt{X}\leftarrow\texttt{a}, \texttt{Y}\leftarrow\texttt{f(b)}, \texttt{Z}\leftarrow\texttt{V}\} = \texttt{q(a,f(b))}$$
$$\texttt{q(X,X)}\{\texttt{X}\leftarrow\texttt{a}, \texttt{Y}\leftarrow\texttt{f(b)}, \texttt{Z}\leftarrow\texttt{V}\} = \texttt{q(a,a)}$$
$$\texttt{q(X,W)}\{\texttt{X}\leftarrow\texttt{a}, \texttt{Y}\leftarrow\texttt{f(b)}, \texttt{Z}\leftarrow\texttt{V}\} = \texttt{q(a,W)}$$
$$\texttt{q(Z,V)}\{\texttt{X}\leftarrow\texttt{a}, \texttt{Y}\leftarrow\texttt{f(b)}, \texttt{Z}\leftarrow\texttt{V}\} = \texttt{q(V,V)}$$

Cascaded Substitutions

$$\mathtt{r(X,Y,Z)}\{\mathtt{x}\leftarrow\mathtt{a},\ \mathtt{y}\leftarrow\mathtt{f(U)},\ \mathtt{Z}\leftarrow\mathtt{V}\} = \mathtt{r(a,f(U),V)}$$

$$\mathtt{r(a,f(U),V)}\{\mathtt{U}\leftarrow\mathtt{d},\ \mathtt{V}\leftarrow\mathtt{e},\ \mathtt{Z}\leftarrow\mathtt{g}\} = \mathtt{r(a,f(d),e)}$$

$$\mathtt{r(X,Y,Z)}\{\mathtt{X}\leftarrow\mathtt{a},\ \mathtt{Y}\leftarrow\mathtt{f(d)},\ \mathtt{Z}\leftarrow\mathtt{e},\ \mathtt{U}\leftarrow\mathtt{d},\ \mathtt{V}\leftarrow\mathtt{e}\} = \mathtt{r(a,f(d),e)}$$

Composition of Substitutions

The *composition* of substitution σ and τ is the substitution (written *compose(*σ,τ*)* or, more simply, στ) obtained by
(1) applying τ to the replacements in σ
(2) adding to σ pairs from τ with different variables
(3) deleting any assignments of a variable to itself.

```
{X←a, Y←U, Z←V}{U←d, V←e, Z←g}
   = {X←a, Y←d, Z←e}{U←d, V←e, Z←g}
   = {X←a, Y←d, Z←e, U←d, V←e}
```

Unification

A substitution σ is a *unifier* for an expression φ and an expression ψ if and only if φσ=ψσ.

$$p(X,Y)\{X\leftarrow a, Y\leftarrow b, V\leftarrow b\} = p(a,b)$$
$$p(a,V)\{X\leftarrow a, Y\leftarrow b, V\leftarrow b\} = p(a,b)$$

If two expressions have a unifier, they are said to be *unifiable*. Otherwise, they are *nonunifiable*.

```
p(X,X)
p(a,b)
```

Non-Uniqueness of Unification

Unifier 1:

$$\texttt{p(X,Y)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{b},\ \texttt{V}\leftarrow\texttt{b}\} = \texttt{p(a,b)}$$
$$\texttt{p(a,V)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{b},\ \texttt{V}\leftarrow\texttt{b}\} = \texttt{p(a,b)}$$

Unifier 2:

$$\texttt{p(X,Y)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{f(W)},\ \texttt{V}\leftarrow\texttt{f(W)}\} = \texttt{p(a,f(W))}$$
$$\texttt{p(a,V)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{f(W)},\ \texttt{V}\leftarrow\texttt{f(W)}\} = \texttt{p(a,f(W))}$$

Unifier 3:

$$\texttt{p(X,Y)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{V}\} = \texttt{p(a,V)}$$
$$\texttt{p(a,V)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{V}\} = \texttt{p(a,V)}$$

Most General Unifier

A substitution σ is a *most general unifier* (*mgu*) of two expressions if and only if it is *as general as or more general than* any other unifier.

Theorem: If two expressions are unifiable, then they have an mgu that is unique up to variable permutation.

$$\texttt{p(X,Y)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{V}\} = \texttt{p(a,V)}$$
$$\texttt{p(a,V)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{Y}\leftarrow\texttt{V}\} = \texttt{p(a,V)}$$

$$\texttt{p(X,Y)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{V}\leftarrow\texttt{Y}\} = \texttt{p(a,Y)}$$
$$\texttt{p(a,V)}\{\texttt{X}\leftarrow\texttt{a},\ \texttt{V}\leftarrow\texttt{Y}\} = \texttt{p(a,Y)}$$

Unification Procedure

One good thing about our language is that there is a simple and inexpensive procedure for computing a most general unifier of any two expressions if it exists.

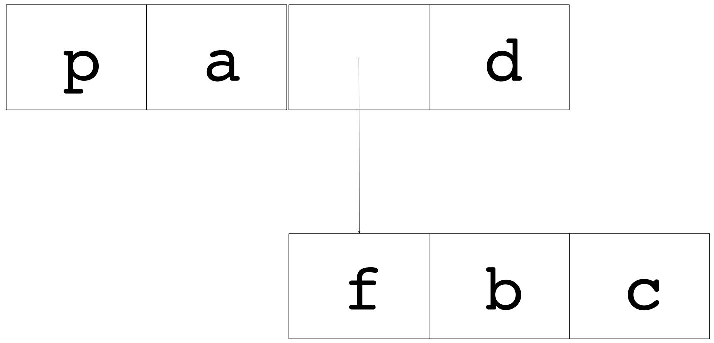
Expression Structure

Each expression is treated as a sequence of its immediate subexpressions.

Linear Version:

```
p(a, f(b, c), d)
```

Structured Version:



Unification Procedure

(1) If two expressions being compared are identical, succeed.

(2) If neither is a variable and at least one is a constant, fail.

(3) If one of the expressions is a variable, proceed as described shortly.

(4) If both expressions are sequences, iterate across the expressions, comparing each subexpression as described above.

Dealing With Variables

If one of the expressions is a variable, check whether the variable has a binding in the current substitution.

(a) If so, try to unify the *binding* with the other expression.

(b) If no binding, check whether the other expression contains the variable. If the variable occurs within the expression, fail. Otherwise, set the substitution to the composition of the old substitution and a new substitution in which variable is bound to the other expression.

Example

Call: `p(X,b)`, `p(a,Y)`, $\{\}$

> **Call:** `p`, `p`, $\{\}$
> **Exit:** $\{\}$
>
> **Call:** `X`, `a`, $\{\}$
> **Exit:** $\{\}\{X \leftarrow a\} = \{X \leftarrow a\}$
>
> **Call:** `b`, `Y`, $\{X \leftarrow a\}$
> **Exit:** $\{X \leftarrow a\}\ \{Y \leftarrow b\} = \{X \leftarrow a, Y \leftarrow b\}$

Exit: $\{X \leftarrow a, Y \leftarrow b\}$

Example

Call: `p(X,X)`, `p(a,Y)`, {}

> **Call:** `p`, `p`, {}
> **Exit:** {}
>
> **Call:** `X`, `a`, {}
> **Exit:** {}{X←a} = {X←a}
>
> **Call:** `X`, `Y`, {X←a}
>> **Call:** `a`, `Y`, {X←a}
>> **Exit:** {X←a} {Y←a} = {X←a, Y←a}
>
> **Exit:** {X←a, Y←a}

Exit: {X←a, Y←a}

Example

Call: `p(X,X)`, `p(a,b)`, {}

Call: `p`, `p`, {}
Exit: {}

Call: `X`, `a`, {}
Exit: {}{$X \leftarrow a$} = {$X \leftarrow a$}

Call: `X`, `b`, {$X \leftarrow a$}
Call: `a`, `b`, {$X \leftarrow a$}
Exit: *false*
Exit: *false*

Exit: *false*

Example

Call: `p(X,X)`, `p(Y,f(Y))`, {}

  Call: `p`, `p`, {}
  Exit: {}

  Call: `X`, `Y`, {}
  Exit: {}{`X`←`Y`} = {`X`←`Y`}

  Call: `X`, `f(Y)`, {`X`←`Y`}
    Call: `Y`, `f(Y)`, {`X`←`Y`}
    Exit: *false*
  Exit: *false*

Exit: *false*

Reason

Circularity Problem:

$$\{X \leftarrow f(Y), Y \leftarrow f(Y)\}$$

Unification Problem:

$$p(X,X)\{X \leftarrow f(Y), Y \leftarrow f(Y)\} = p(f(Y),f(Y))$$

$$p(Y,f(Y))\{X \leftarrow f(Y), Y \leftarrow f(Y)\} = p(f(Y),f(f(Y)))$$

Before assigning a variable to an expression, first check that the variable does not occur within that expression.

This is called the *occur check* test.

Prolog does not do the occur check (and is proud of it).
But it can give incorrect answers as a result.

General Goals and Rules

Procedure With Variables

Procedure without variables uses *equality* tests.

```
p(a,b)
p(b,c)
s(a,c) :- p(a,b) & p(b,c)

s(a,c)?
```

Procedure with variables uses *unification*.

```
p(a,b)
p(b,c)
s(X,Z) :- p(X,Y) & p(Y,Z)

s(a,c)?
```

Step 1 - Atoms with Base Relations

Given an atom with a base relation and a substitution:

(a) Compare the goal to each factoid in our dataset.

(b) If there is an extension of the given substitution that unifies the goal and the factoid, add to our list of answers.

(c) Once all relevant factoids examined, return answers.

Example 1 - Atoms with Base Relations

Goal: `p(X,Y)`

Substitution: {X←a}

Dataset: {p(a,b),p(a,c),p(b,c)}

Result: [{X←a, Y←b}, {X←a, Y←c}]

Step 2 - Negations

Given a negation and a substitution:

(a) Execute the procedure on the target of the negation and the given substitution.

(b) If the result is empty, return a singleton list containing the given substitution, indicating success.

(c) Otherwise, return the empty list of answers, indicating failure.

Example 2 - Negations

Goal: `~p(X,Y)`
Substitution: {`X`←`a`, `Y`←`d`}
Dataset: {`p(a,b)`, `p(a,c)`, `p(b,c)`}
Result: [{`X`←`a`, `Y`←`d`}]

Goal: `~p(X,Y)`
Substitution: {`X`←`a`, `Y`←`c`}
Dataset: {`p(a,b)`, `p(a,c)`, `p(b,c)`}
Result: []

Step 3 - Conjunctions

Given a conjunction and a substitution:

(a) Execute our procedure on the first conjunct and the given substitution to get a list of answers.

(b) Iterate through the list of substitutions, calling the procedure recursively on the remaining conjuncts with each substitution in turn.

(c) Collect the answers from recursive calls and return.

Example 3 - Conjunctions

Goal: `p(X,Y) & p(Y,Z)`
Substitution: $\{X \leftarrow a\}$
Dataset: `{p(a,b),p(a,c),p(b,c)}`

Call: `p(X,Y)`, $\{X \leftarrow a\}$
Result: [$\{X \leftarrow a, Y \leftarrow b\}$,$\{X \leftarrow a, Y \leftarrow c\}$]

Call: `p(Y,Z)`, $\{X \leftarrow a, Y \leftarrow b\}$
Result: [$\{X \leftarrow a, Y \leftarrow b, Z \leftarrow c\}$]

Call: `p(Y,Z)`, $\{X \leftarrow a, Y \leftarrow c\}$
Result: []

Overall Result: [$\{X \leftarrow a, Y \leftarrow b, Z \leftarrow c\}$]

Step 4 - Atoms with View Relations

Given atom with view relation and a substitution:

(a) Iterate through the rules in our program.

(b) *Copy each rule, replacing variables with new variables.*

(c) Try to unify the given goal and the new rule head.

(d) Call the procedure recursively on the body of the rule.

(e) Return substitutions from all successful cases.

Example 4 - Atoms with View Relations

Goal: `q(X,Y)`
Substitution: `{X←a}`
Rule: `q(X,Z) :- p(X,Y) & p(Y,Z)`
Dataset: `{p(a,b),p(a,c),p(b,c)}`

Copy of rule: `q(U,W) :- p(U,V) & p(V,W)`

Unification: `q(U,W) q(X,Y) {X←a}`
Result: `{U←a,W←Y,X←a}`

New Goal: `p(U,V) & p(V,W)`
New Substitution: `{U←a,W←Y,X←a}`

Result: `[{U←a,W←c,X←a,V←b,Y←c}]`

Compound Terms

Compound terms compound the difficulty.

Rule

```
s(X,f(Y,Z)) :- p(X,g(Y)) & p(Y,X)
```

Query

```
s(h(X),X)
```

Subgoal

```
p(h(f(Y,Z)),g(Y)) & p(Y,h(f(Y,Z)))
```

Efficiency Enhancements

Multiple substitutions

Different substitutions used for goals and rules

Good: Rules are not copied

Evaluation of conjuncts is pipelined

Once each answer to a conjunct is computed,
the other conjuncts are checked immediately;
then other answers generated and checked.

Good: Saves work when only few answers needed.

Good: Avoids problems due to infinite answer sets.

Upshot: This is complicated. Don't try this at home. Leave it to the professionals.

Tracing

Facts and Rules

```
p(a,b)
p(b,c)
s(X,Z) :- p(X,Y) & p(Y,Z)
```

Trace

```
Call: s(X,Z)
| Call: p(X,Y)
| Exit: p(a,b)
| Call: p(b,Z)
| Exit: p(b,c)
Exit: s(a,c)
```

Backup Tracing

Facts and Rules

```
p(a,b)
p(b,c)
s(X,Z) :- p(X,Y) & p(Y,Z)
```

Trace

```
Call: s(X,Z)
| Call: p(X,Y)
| Exit: p(a,b)
| Call: p(b,Z)
| Exit: p(b,c)
Exit: s(a,c)
```

```
Redo: s(X,Z)
| Redo: p(b,Z)
| Fail: p(b,Z)
| Redo: p(X,Y)
| Exit: p(b,c)
| Call: p(c,Z)
| Fail: p(c,Z)
Fail: s(X,Z)
```

Summary

Comparison of Evaluation Strategies

Bottom-Up Evaluation

Easy to understand
Computes all results
Computes subresults just once
Wasteful when want just one or a few answers, not all
Problematic on logic programs with infinite models

Top-Down Evaluation

Less waste when want one or a few answers
More complicated
Subqueries evaluated multiple times
Possibility of infinite loops on programs w/ finite models

But …

Bottom-Up Evaluation

Can be focussed using Magic Sets

Top-Down Evaluation

Top-Down can avoid duplication through caching

Infinite Loops can be avoided using iterative deepening

The arms race continues.

Sierra

Sierra

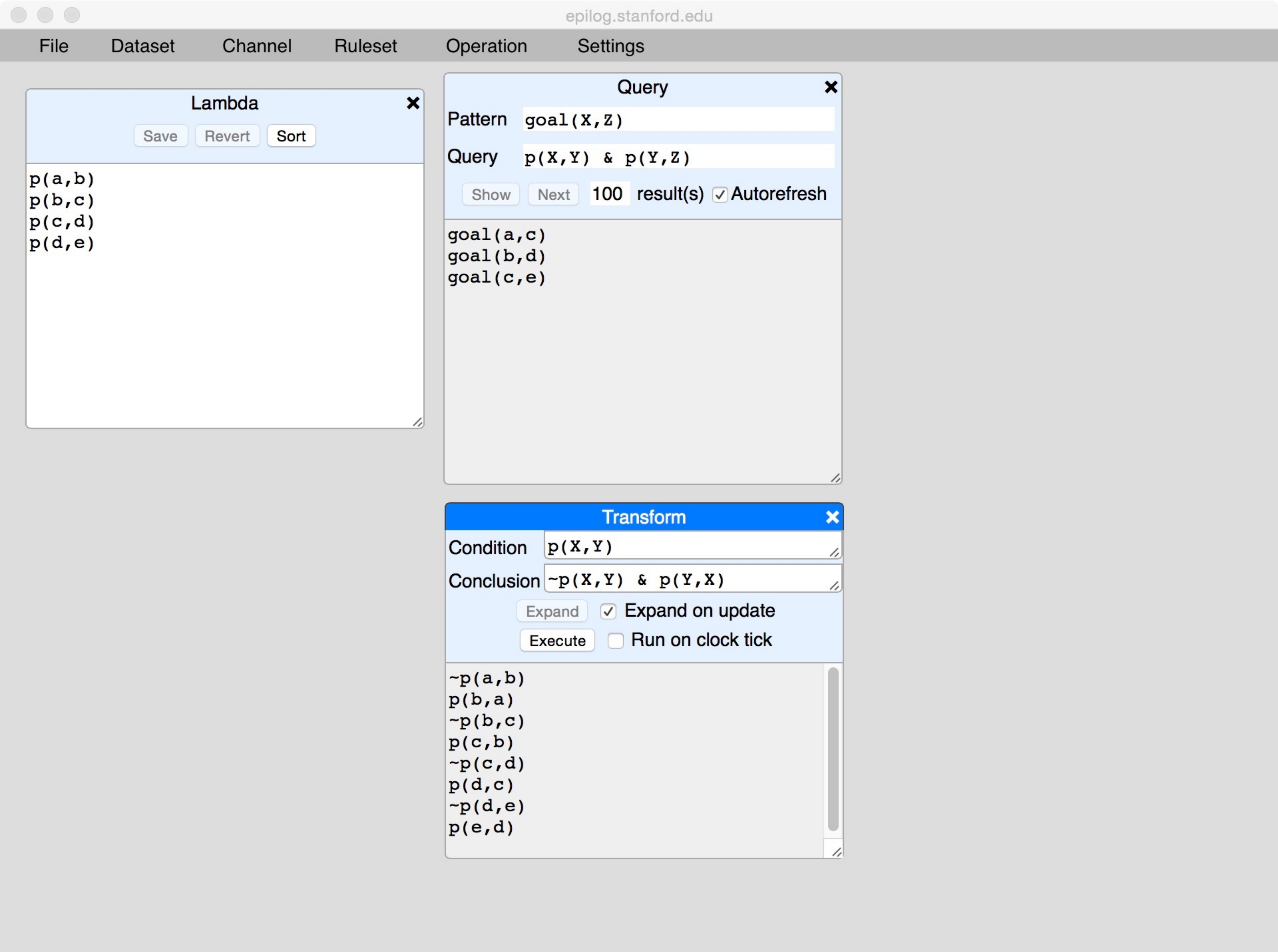
Sierra is browser-based IDE (interactive development environment) for Epilog.

Saving and loading files

Viewing and Editing datasets
Querying datasets
Transformation tools for datasets

Interpreter (for view definitions, action definitions)
Trace capability (useful for debugging rules)
Analysis tools (error checking and optimizing rules)

http://epilog.stanford.edu/homepage/sierra.php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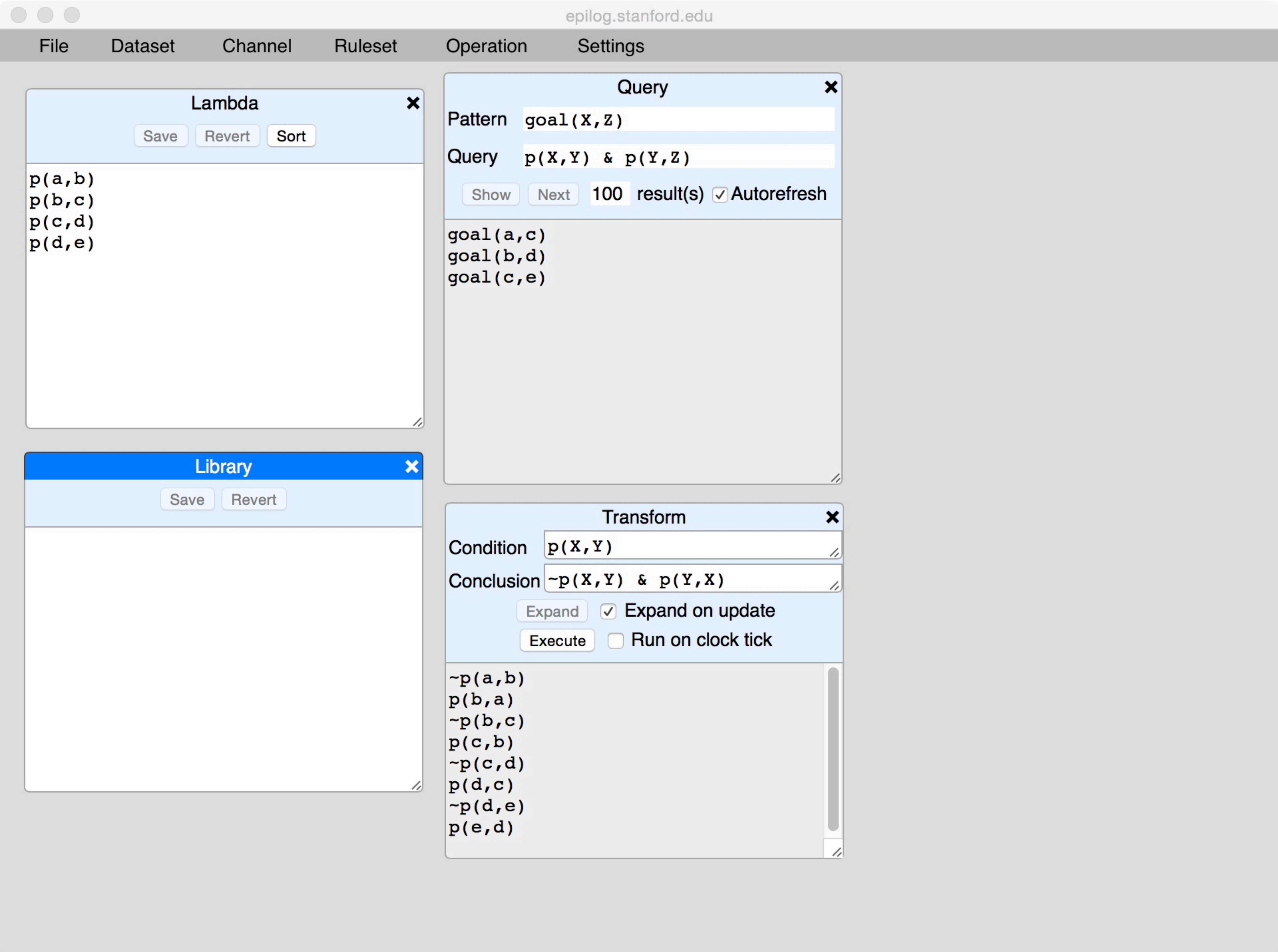
File
Dataset
Channel
Ruleset
Operation
Settings
Lambda
Save
Revert
Sort
p(a,b)
p(b,c)
p(c,d)
p(d,e)
Query
Pattern
goal(X,Z)
Query
p(X,Y) & p(Y,Z)
Show
Next
100
result(s)
Autorefresh
goal(a,c)
goal(b,d)
goal(c,e)
Transform
Condition
p(X,Y)
Conclusion
~p(X,Y) & p(Y,X)
Expand
Expand on update
Execute
Run on clock tick
~p(a,b)
p(b,a)
~p(b,c)
p(c,b)
~p(c,d)
p(d,c)
~p(d,e)
p(e,d)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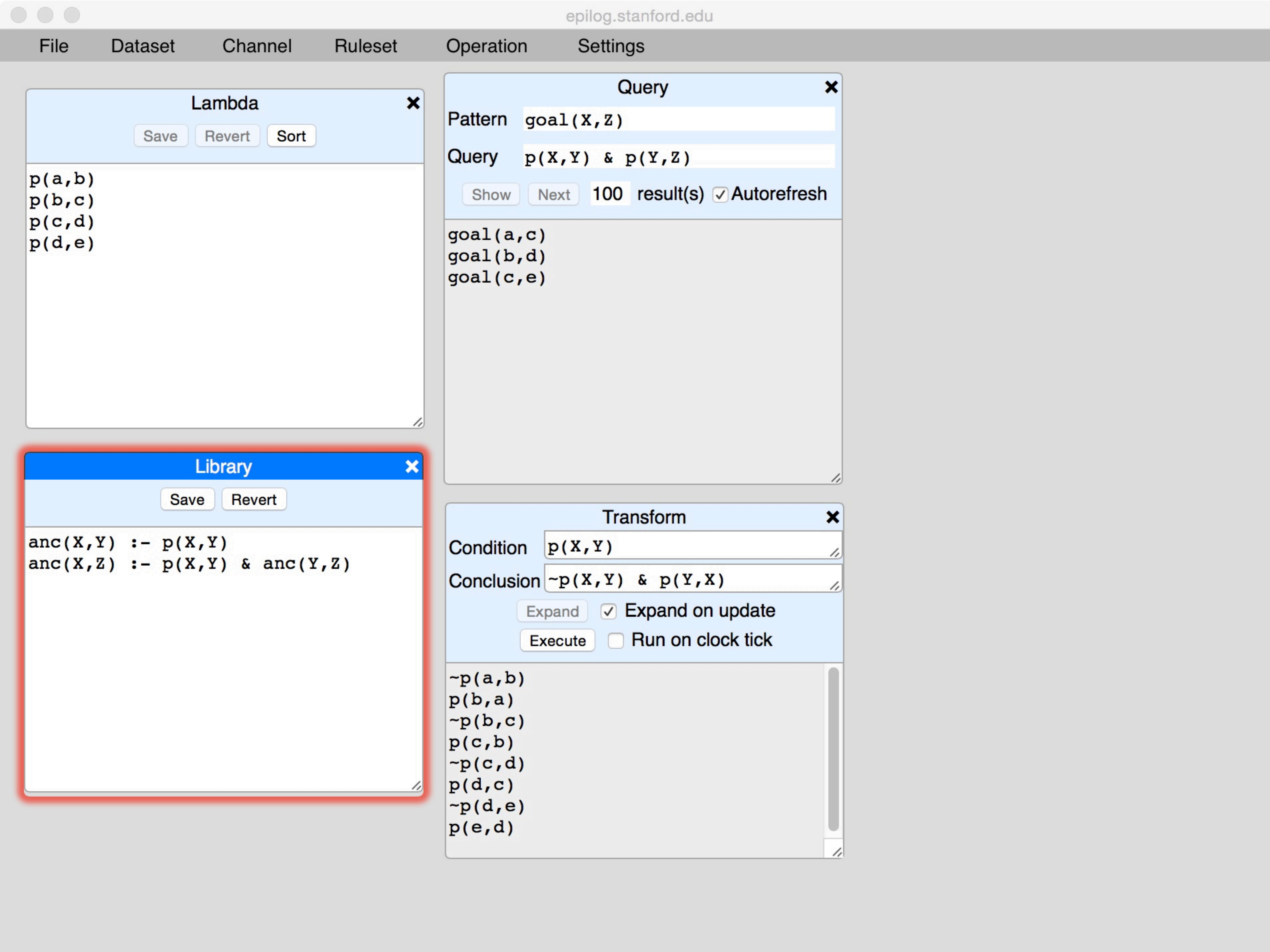
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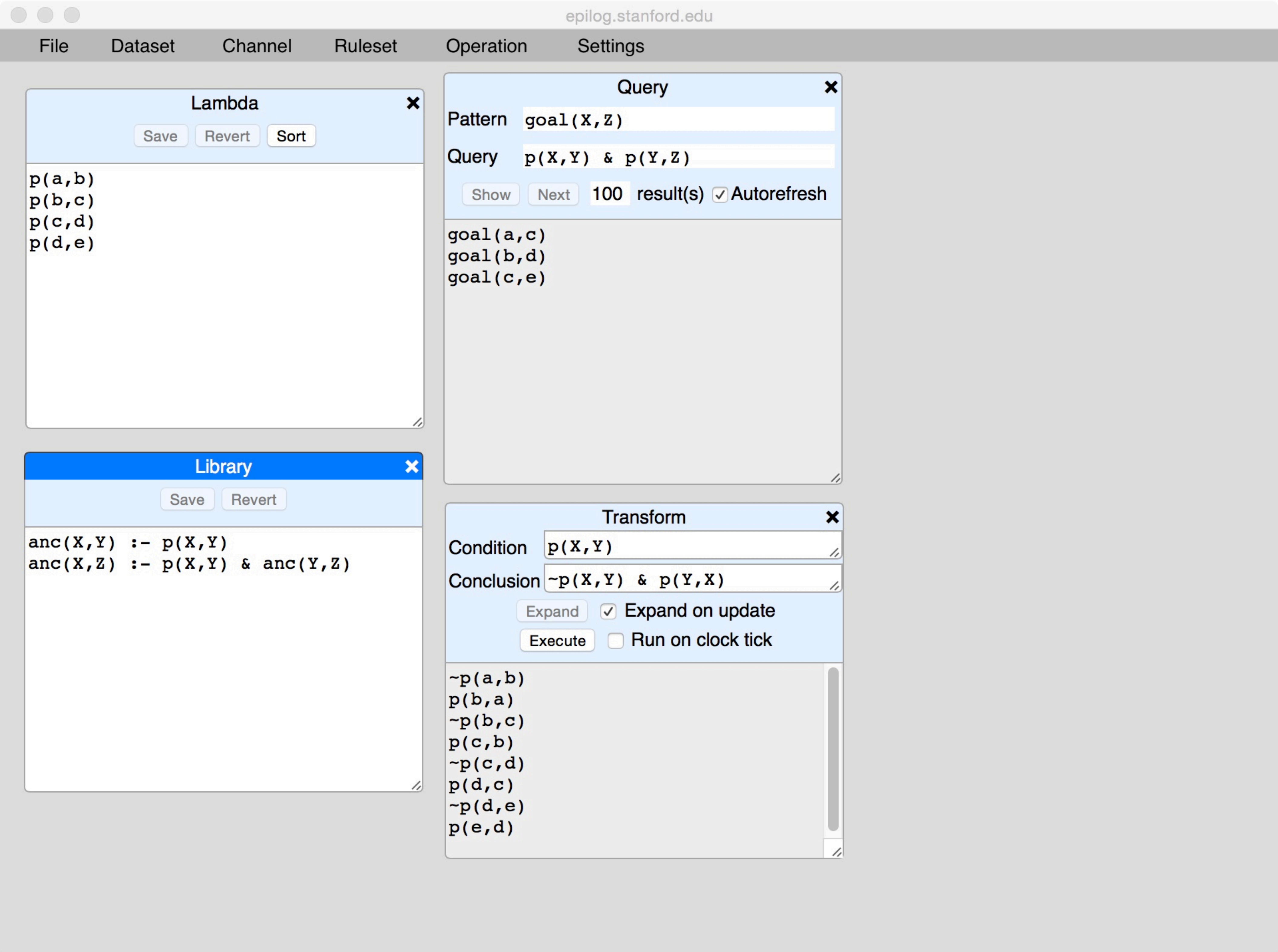
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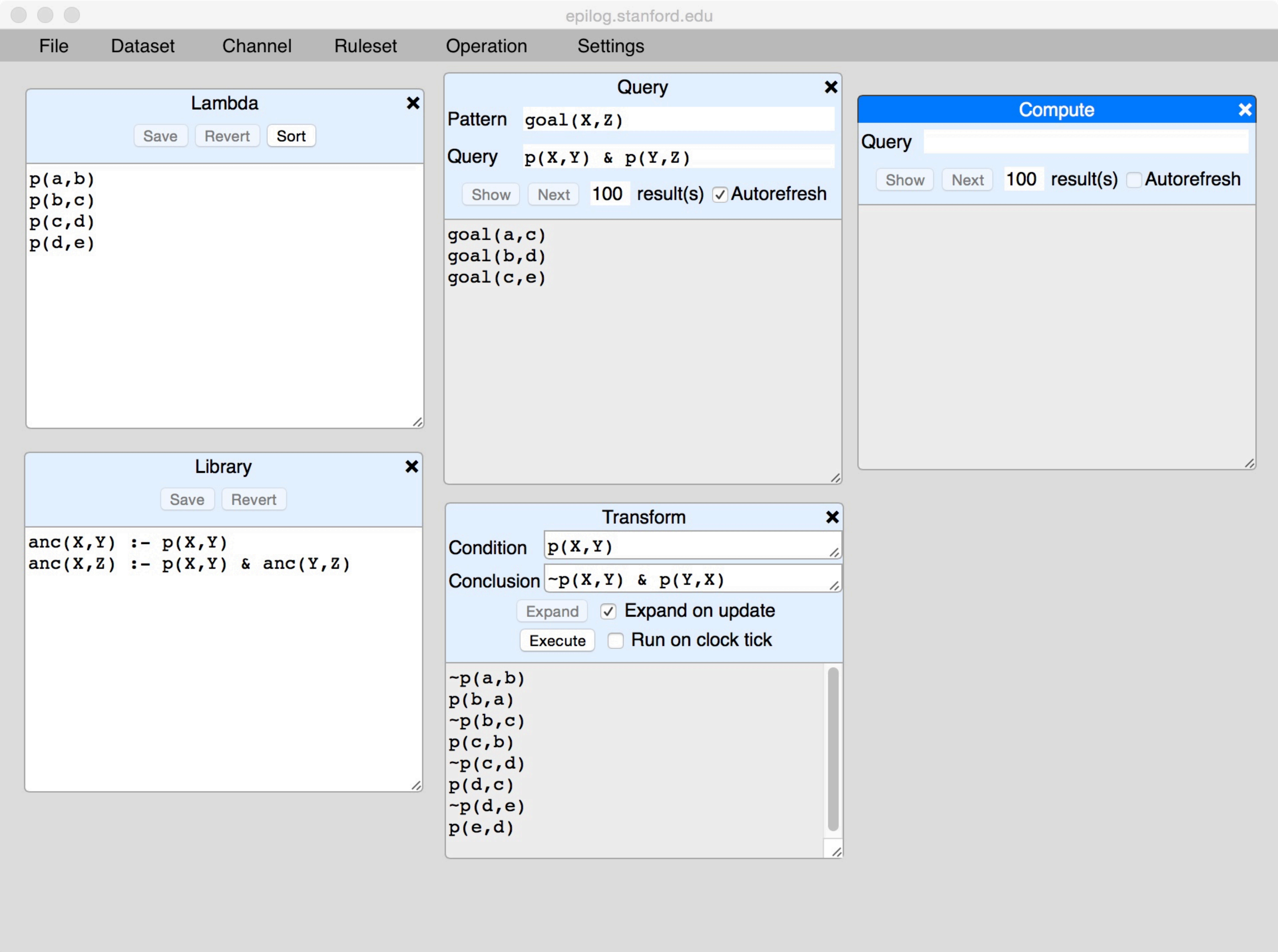
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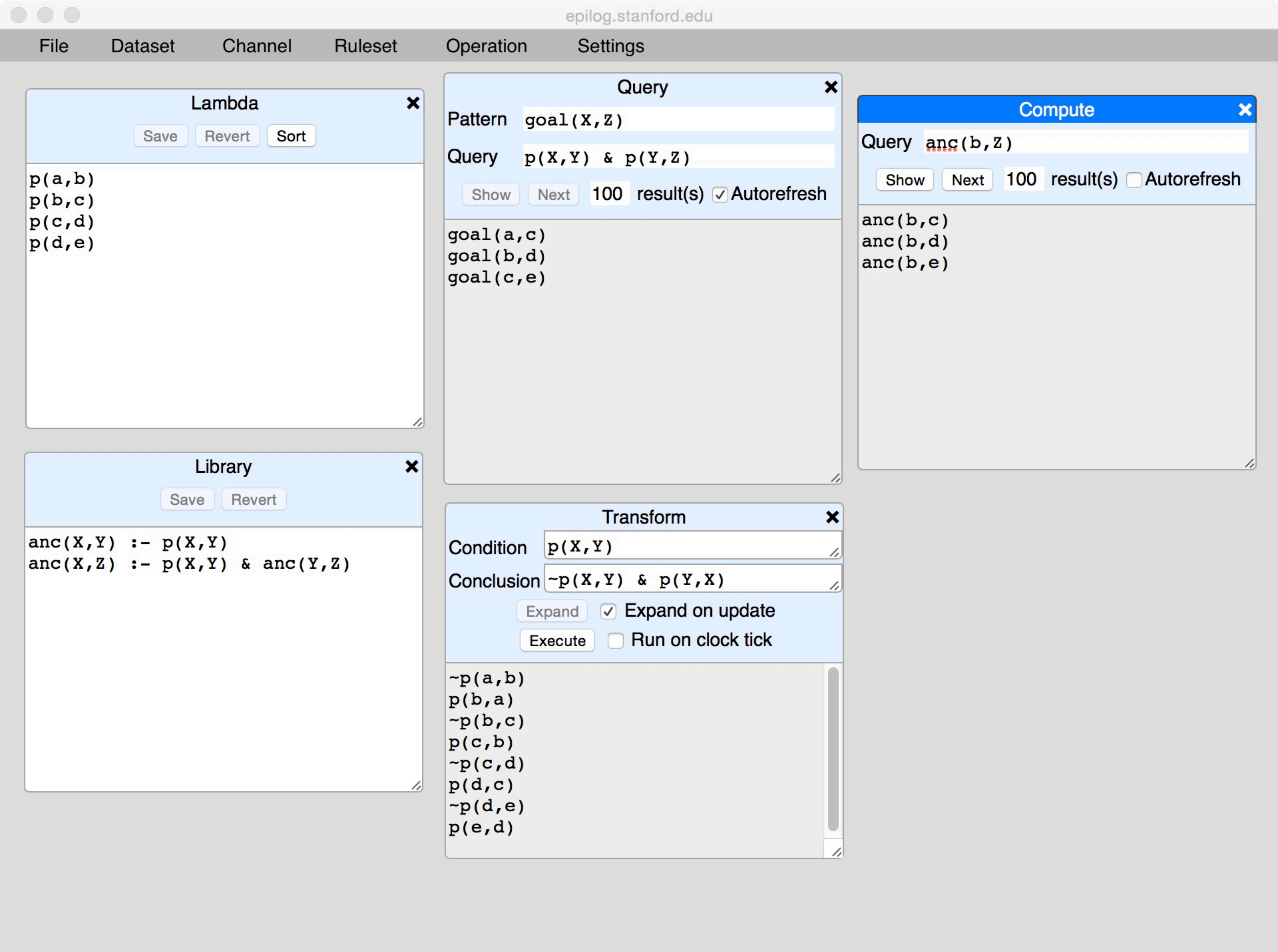
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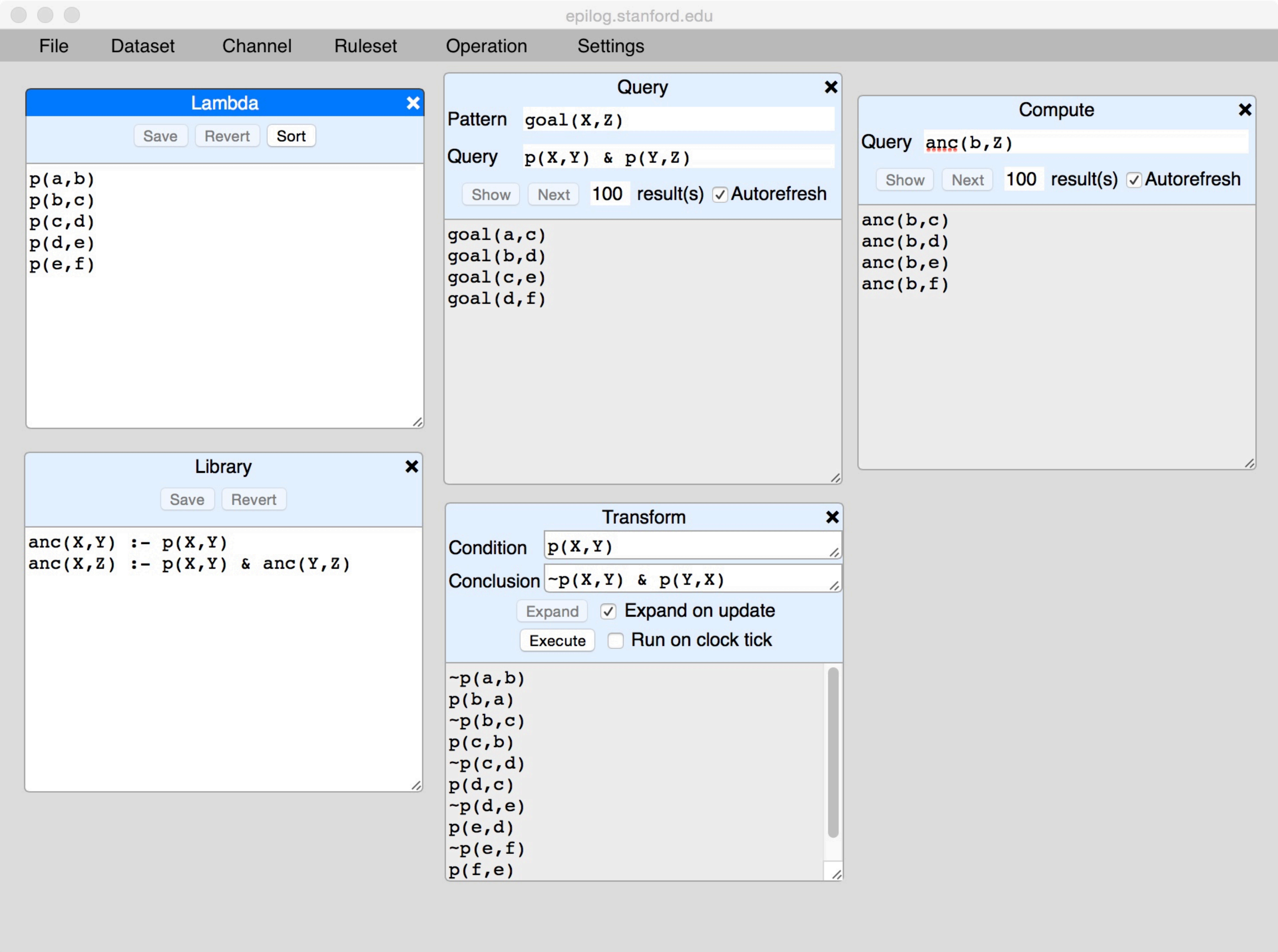
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p(c,d)
p(d,e)
p(e,f)
Library
Save
Revert
anc(X,Y) :- p(X,Y)
anc(X,Z) :- p(X,Y) & anc(Y,Z)
Query
Pattern goal(X,Z)
Query p(X,Y) & p(Y,Z)
Show
Next
100 result(s)
Autorefresh
goal(a,c)
goal(b,d)
goal(c,e)
goal(d,f)
Transform
Condition p(X,Y)
Conclusion ~p(X,Y) & p(Y,X)
Expand
Expand on update
Execute
Run on clock tick
~p(a,b)
p(b,a)
~p(b,c)
p(c,b)
~p(c,d)
p(d,c)
~p(d,e)
p(e,d)
~p(e,f)
p(f,e)
Compute
Query anc(b,Z)
Show
Next
100 result(s)
Autorefresh
anc(b,c)
anc(b,d)
anc(b,e)
anc(b,f)